

OPTIMAL DESIGN OF PRESTRESSING CABLE IN PRECAST TWO SPAN CONTINUOUS BEAMS BY MOMENT BALANCING METHOD

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

BY
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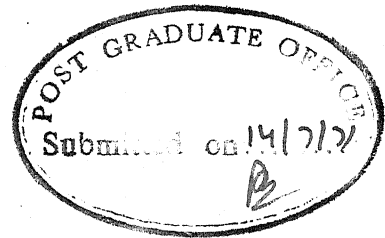
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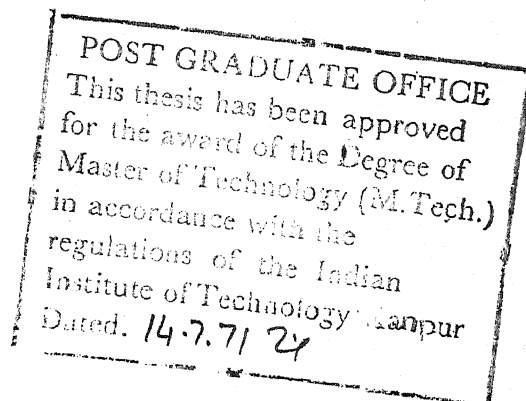
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CERTIFICATE

This is to certify that the thesis entitled "Optimal Design of Prestressing Cable in Precast Two Span Continuous Beams by Moment Balancing Method" by Putcha Chandrasekhar is record of work carried out under my supervision and has not been submitted elsewhere for a degree.

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TABLE OF CONTENTS

CHAPTER		PAGE
I.	INTRODUCTION AND STATEMENT OF THE PROBLEM	
	1.1 Precast Construction in Bridges	1
	1.2 Statement of the Problem	4
II.	FORMULATION OF THE PROBLEM	
	2.1 Load Conditions	7
	2.2 Idealization of Load Conditions for Effective Bending Moment	11
	2.3 Balancing Moment	12
	2.4 Continuity Compatibility	26
III.	DESIGN OF PRESTRESSING CABLE	
	3.1 Design Bending Moments	31
	3.2 Minimization of the Unbalance Bending Moment	32
	3.3 Normalization of Data	35
	3.4 Results	37
	3.5 Design Example	40
	3.6 Discussion of Results	51
	3.7 Conclusions	58
	REFERENCES	59

LIST OF TABLES

TABLE		PAGE
2.1	Load Conditions of Set 2	11
2.2	Moment Distribution for q_2	23
2.3	Moment Distribution for the Concentrated Prestressing force at Anchorage	24
2.4	Moment Distribution for eccentrically acting horizontal force	25
3.1.	Moment Distribution for Class A loading for antisymmetric case	41
3.2	Moment Distribution for Class A loading for symmetric case	48

LIST OF FIGURES

FIGURE		PAGE
1.1	B.M.D. before and after redistribution of moments	3
1.2	External Load Condition	5
2.1	Beam with Load Condition 1	8
2.2	Beam with Load Condition 2	9
2.3	Beam with Load Condition 3	10
2.4	Idealization of Live Loads for qualitative study	13
2.5	Cable Profiles	14
2.6	Equivalent Balancing Forces	15
2.7	Balancing Force and B.M.D. Due to Cable Set 1	16
2.8	Continuity Cable	18
2.9	Balancing Force and B.M.D. Due to Cable Set 2	22
2.10	Slope Matching Due to Continuity Cable	28
2.11	Slope due to the parabolic Cable and the effect of loss due to Prestressing	29
3.1	Typical Unbalance B.M.D. For a Load Condition	33
3.2	Flow Chart for the Computer Programme used	36
3.3	Design Bending Moment Diagram ($g=0.55$ & $\omega=0.25$)	38
3.4	Design Bending Moment Diagram ($g=0.60$ & $\omega=0.20$)	39
3.5	Idealization of net Bending Moment Diagram by an equivalent u.d.l. for antisymmetric case	42
3.6	Prestressing force and the unbalance B.M. diagrams for antisymmetric loading	44
3.7	Idealization of net B.M.D. by an equivalent u.d.l. for symmetric case	49
3.8	Design B.M.D. and section of the beam	50
3.9	Effect of g - Constant ω and r ($\omega=0.15$ & $r=1.5$)	53
3.10	Effect of g - Constant ω and r ($\omega=0.25$ & $r=1.5$)	54
3.11	Effect of ω - Constant g and r	55
3.12	Selection of Prestress (for fixed values of g)	56
3.13	Selection of Prestress (for fixed values of ω)	57

LIST OF SYMBOLS & ABBREVIATIONS

- q_1 = live load on the beam
 q_g = self weight of the beam
 P_1 = prestressing force in the simply supported cable
 P_2 = prestressing force in the continuity cable
 M_{xs} = B.M. at any section due to the simply supported cable
 M_{xc} = B.M. at any section due to the continuity cable
 αL = length of the curve 2 on either side of the middle support till the common point.
 βh = height of the curve 2 at middle support from the bottom fibre
 γh = $(\beta - 0.5)h$
 ωL = $1/8 \frac{\alpha^2}{\gamma} L = \text{span of the continuity cable}$
 g_h = sag of the simply supported cable
 e_h = eccentricity of the simply supported cable
 r = the ratio of the B.M. due to live load and self weight
 M_{ubl}^i = unbalance bending moment at i^{th} point for 1^{th} load condition
 C_{p1}^i = coefficient of the bending moment due to prestressing force at i^{th} point for 1^{th} load condition
 C_{g1}^i = coefficient of bending moment due to external load (expressed in terms of $q_g L^2$) at i^{th} point for 1^{th} load condition.
 $= P_2 / q_g L^2$
 M_1 = bending Moment due to live load
 M_g = bending Moment due to self weight
 P_{1e} = prestressing force at working condition in simply supported cable

- P_{2e} = Prestressing force at working condition in the continuity cable
- r_1 = ratio of the Prestressing forces
- f'_c = maximum permissible stress in compression
- A = Gross sectional area
- f_{ct} = stress at transfer in compression
- f_{ce} = stress at working in compression
- f_{tt} = stress at transfer in tension
- f_{te} = stress at working in tension
- ρ = efficiency ratio
- Δ = shape factor
- r_g = radius of gyration of the section
- y_b = distance of N.A. from the bottom flange
- y_t = distance of N.A. from the top flange
- B.M. = Bending moment
- N.A. = neutral axis

CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

1.1 Precast construction in Bridges :

Precast methods are well adopted for long span prestressed concrete bridges. Prestressed concrete is economical for long span bridges. For obtaining the overall economy, feasibility of erection and low cost of construction must also be achieved.

Many bridges were built employing the precast techniques. Some of such bridges are :

i) Tasman bridge, Robart, Tasmania, with main spans of 197, 310 and 197 feet, rising to a height of 145 feet above sea level.

ii) The bridge across the Moscow river in the vicinity of an automobile plant at Likhachev with a span of 485 feet¹.

iii) Bridge across Danube river at New garden Yugoslavia with prestressed arch spans of 690 feet and 543 feet¹.

iv) Bridge over Barak river near Silchar in the Cachar District of Assam with central span of 400 feet and the span on either side being 185 feet each².

v) Road bridge at Dehri-on-Sone in Bihar. It has 93 spans of 108 feet each and with accommodation of 24 feet wide roadway with 5 feet wide footpaths for pedestrian traffic on either side.

The numerals in the superscripts indicate the references given at the end.

vi) Bridge over the river at Torsha, near Hashimara at the junction of lateral route and NH31 - about 38 miles from Cooch Behar town, in West Bengal. It has 10 spans of average 143 feet and has been designed for single lane class AA loading or double lane class A loading⁴.

vii) Bridge across Godavari at Bhadrachalam. It has a total length of 3,934 feet comprising of 37 spans of 106 feet 6 in each⁵.

In the case of 2 span prestressed precast concrete bridges the beams are precast and prestressed in single span, and made continuous through a continuity cable profile. Providing a single prestressing cable profile throughout the continuous beam would have mainly two disadvantages :

i) The first problem is of handling and transportation to the site.

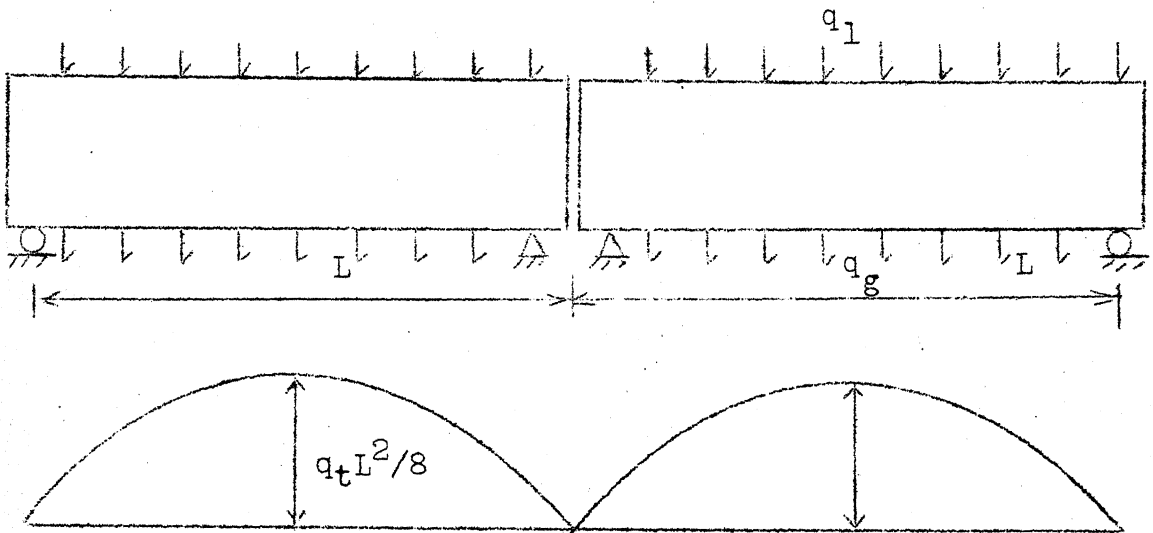
ii) The second problem is prestressing losses would be in substantial.

Making the prestressed precast beams continuous through the continuity cable profile has the following advantages :

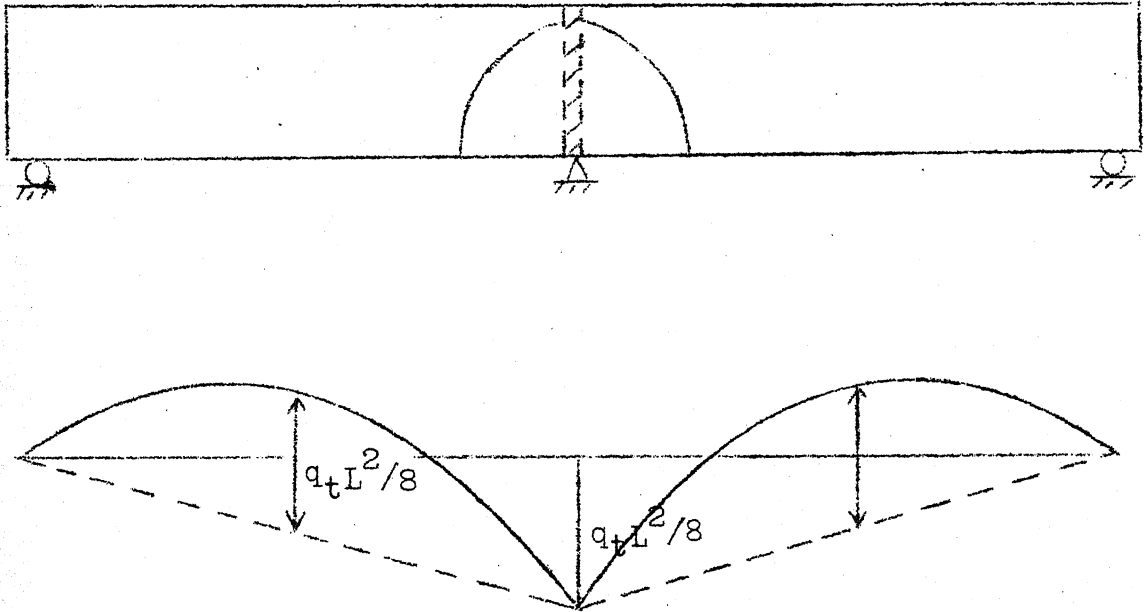
i) It reduces the problem of handling and transportation of long members.

ii) The losses of prestress would be reduced as the length and the included angle of any cable will be much smaller as compared to a single unit two span.

Fig.1.1(a) represents the bending moment diagram before a continuity cable profile is provided. The maximum bending



(a) Simply Supported B.M. diagram
due to External Load



(b) Continuous B.M. Diagram due
to External Load

Fig.1.1 B.M. Diagrams before and after
redistribution of moments

moment is $q_t L^2/8$ where, q_t = total load.

Fig.1.1.(b) represents the bending moment diagram after the continuity cable profile is provided. In this case since the beams are made continuous by means of continuity cable profile redistribution of moment takes place and the magnitude of bending moment at the centre of each beam becomes $q_t L^2/16$ while at the support it would become $q_t L^2/8$ which would be taken care of by the simply supported cable profile, and the continuity cable profile.

In case of foundation settlement problems, continuous beam is not economical since enormous bending moment would be developed due to any unequal settlements.

1.2 Statement of the problem :

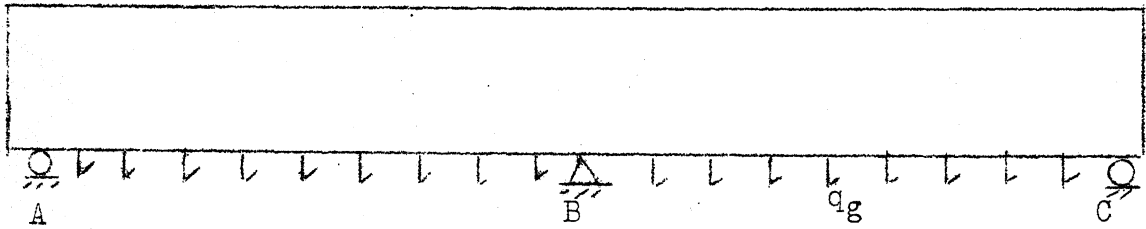
It is assumed that the beams are precast and prestressed in single span and launched to the site. Then two adjacent units are made continuous through a continuity cable profile. Therefore, a continuous beam would have 2 sets of cable profile.

i) Set 1 consists of cables in the single span. These cables are primarily simple parabolas.

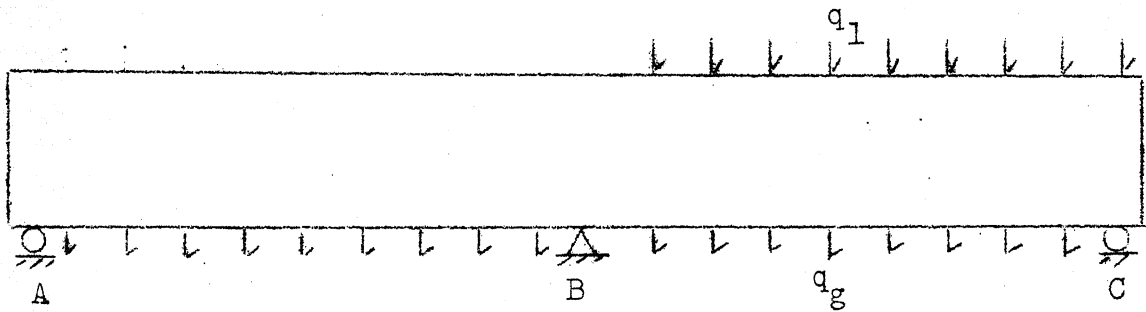
ii) Set 2 correspond to the post tensioned continuity cable profile. The continuity cable is taken as multiple parabolic.

iii) For making the design, 3 design load conditions have been taken into account . They are :

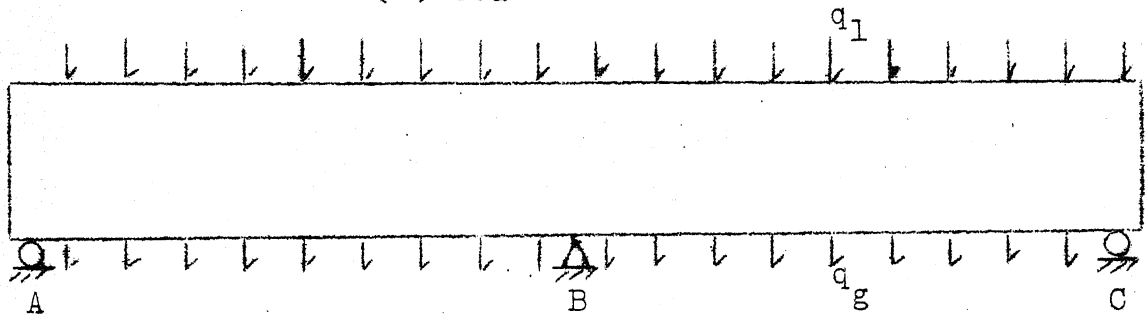
1) First design load condition is when live load is not there and only self weight q_g is acting. Fig. 1.2(a)



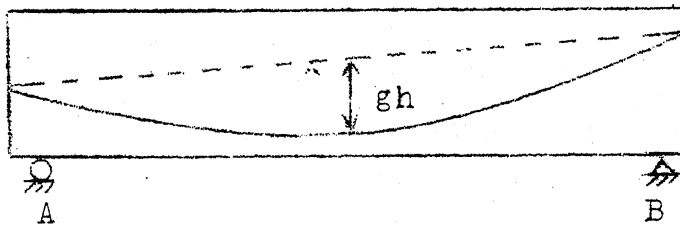
(a) Load Condition 1



(b) Load Condition 2



(c) Load Condition 3



(d) Simply Supported Load Condition

Fig.1.2 External Load Condition

ii) Second design load condition is when live load q_1 is acting only on one of the spans, with q_g acting throughout the continuous beam. Fig.1.2(b)

iii) Third design load condition is when both the self weight q_g and live load q_1 are acting throughout the continuous beam. Fig.1.2(c)

Check for stresses has been made for the 4th load condition.

iv) The fourth load condition is when only one simply supported span is taken into consideration, with only simply supported cable being there. Fig.1.2(d)

✓ Different ratios of dead and live load have been taken into account, and their effect also has been studied on the unbalance bending moment alongwith other effect. The principle of moment balancing⁶ has been used which is just an extension of load balancing✓ which can be stated as follows:

Prestressing balances a certain portion of the total moments due to total load ($q_g + q_1$) so that flexural members such as slabs, beams and girders will not be subjected to complete external bending stresses under a given loading condition.

CHAPTER II

FORMULATION OF THE PROBLEM

2.1 Load Conditions :

Object of the programme is to establish an optimal cable profile in two span continuous beam. There are two sets of load conditions :

Set 1 : Single span handling and erection

Set 2 : Continuous beam with multi load condition.

The design is primarily based on the 2nd set of load conditions and the moments in handling etc. (set 1 loads) are adjusted or checked to be less than the permissible ones. Therefore, primary design load conditions are of Set 2.

Set 2 load conditions are :

i) The beam subjected to self load only (q_g = u.d.l. due to self weight) Fig.2.1.

ii) q_g is acting on both the spans, but q_1 is acting only on any one of the spans (Fig.2.2) where q_1 = live load.

iii) Both q_g and q_1 are assumed to act throughout the continuous beam and assumed to be uniformly distributed. Fig.2.3

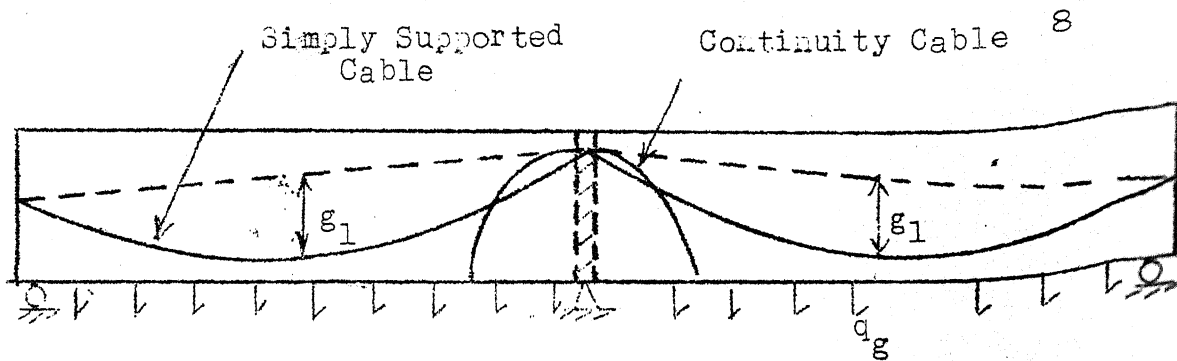
Let $G = q_g L$

$L = q_1 L$

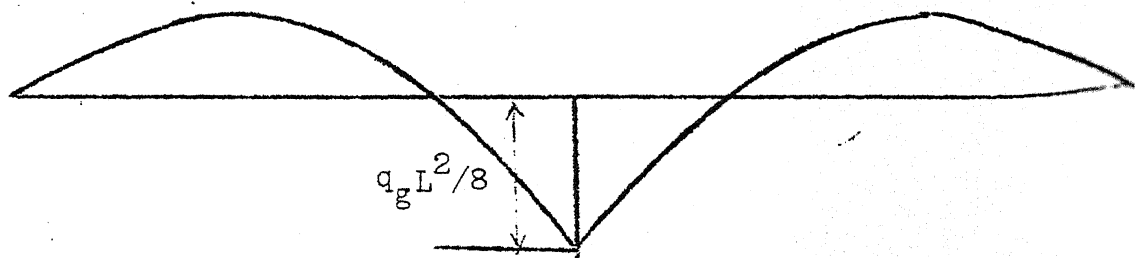
P_1 = Prestressing force in the simple cable (Same in both the spans)

P_2 = Prestressing force in the continuity cable.

Load conditions of set 2 have been illustrated in Table 2.1.

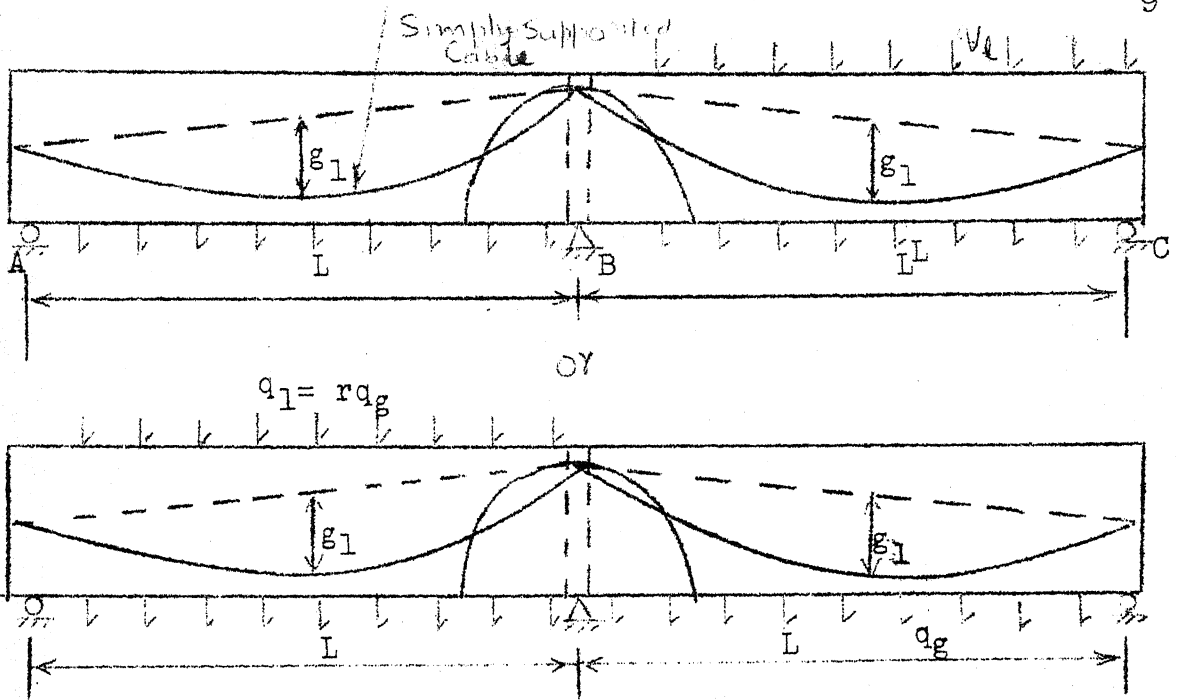


(a) Loads on Beam

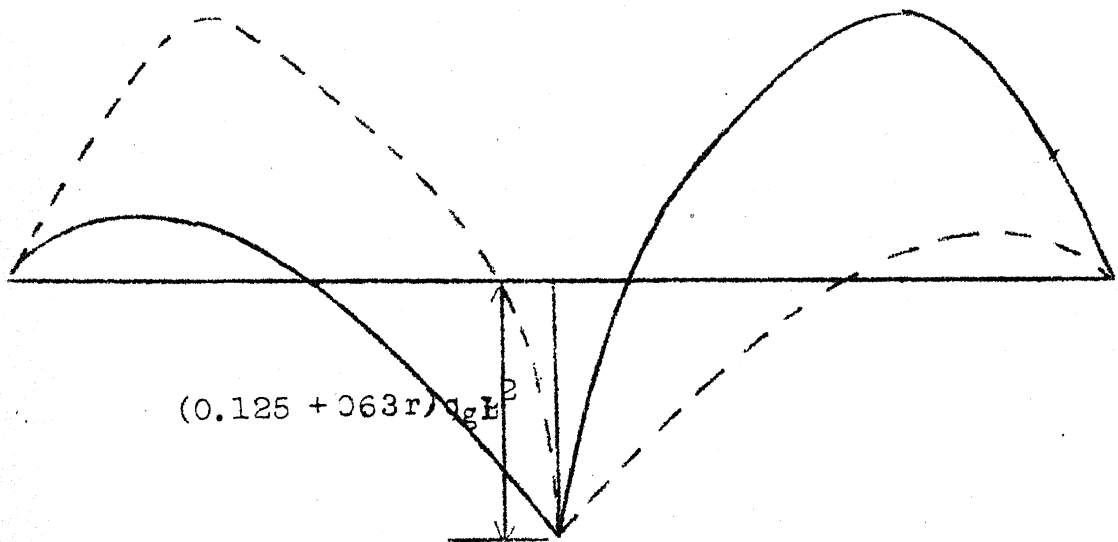


(b) B.M.D. Due to External Load Condition 1

Fig.2.1 Beam with Load Condition 1

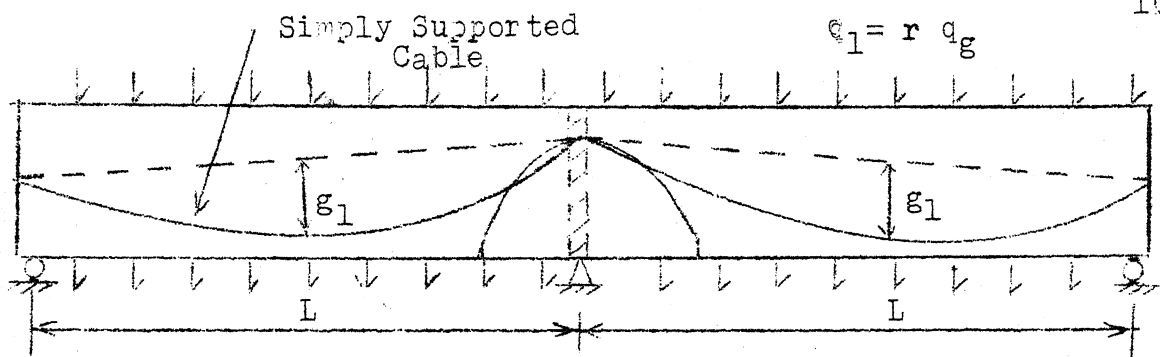


(a) Loads on Beam

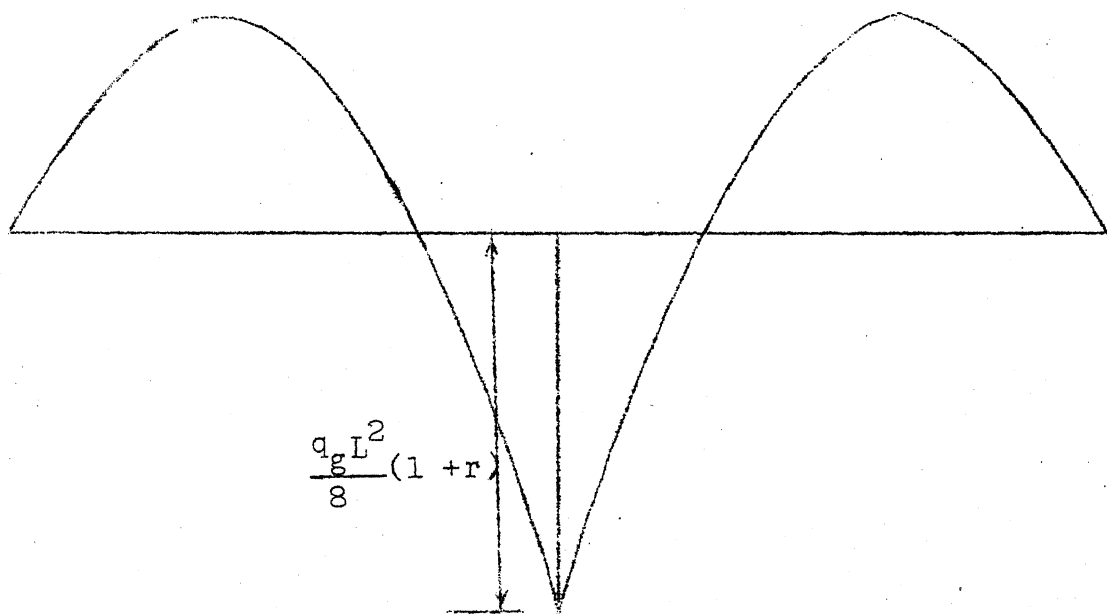


(b) B.M.D. Due to External Load Condition 2

Fig. 2.2 Beam with Load Condition 2



(a) Loads on Beam



(b) B.M.D. Due to External Load Condition 3

Fig.2.3 Beam with Load Condition 3

Making a critical review of all the three bending moment diagrams it can be concluded that the maximum positive bending moment occurs in the case of case 2 loading at $x=0.6L$ and the maximum negative bending

Table 2.1 - Load conditions of Set 2

Load condition	Span A B	Span B C	Remarks
1.	$G + P_1 + P_2$	$G + P_1 + P_2$	symmetric
2. (a)	$G + P_1 + P_2$	$G + P_1 + P_2 + L$	antisymmetric live load
(b)	$G + P_1 + P_2 + L$	$G + P_1 + P_2$	
3.	$G + P_1 + P_2 + L$	$G + P_1 + P_2 + L$	symmetric

moment occurs at $x = 0.0$ in the case of Case II loading.

2.2 Idealization of Load Conditions for Effective Bending

Moment :

The design load specifications either for road or railway bridges are prescribed based on the wheel loads. It would be possible to get the exact bending moments if the span of the bridge is fixed. But it is not that easy even to select the loads (say like whether, Class A or Class A A which one of them dominates) if one has to work with an arbitrary span L . For example governing design load for a two lane national highway of 90 m span is class A while for 40 m span is class AA. Therefore, it would be rather hard to generate a programme as

suggested in this thesis for an arbitrary span in terms of the exact load conditions. An idealization of equivalent distribution is suggested. There could be several types of idealizations, one of the simple and good idealization is that of enveloping parabola. Fig.2.4 illustrates the idealized replacement of the actual bending moment diagram due to the external by an enveloping parabola. The idealized equivalent distributed load is not to be in actual design process but is only used as a tool in development of an optimal design parameters. The parameters are developed from the unbalance bending moment. The influence of idealization of the live load on the unbalance (effective design) bending moment is not appreciable. From Fig.2.4 the magnitude of maximum bending moment can be taken as

$$M_e = q_e L^2 / 8$$

$$\therefore q_e = 8 M_e / L^2 \quad (2.1)$$

2.3 Balancing Moment :

There are two prestressing cables (Fig 2.5)

- a) Parabolic cables for single span
- b) Multiple parabolic continuity Cable.

Both P_1 and P_2 cables should balance a partial external load. The individual effect of Prestressing forces (P_1 and P_2) are computed and then superimposed.

(a) The Parabolic Cable :

Parabolic Cable for single span has been provided as

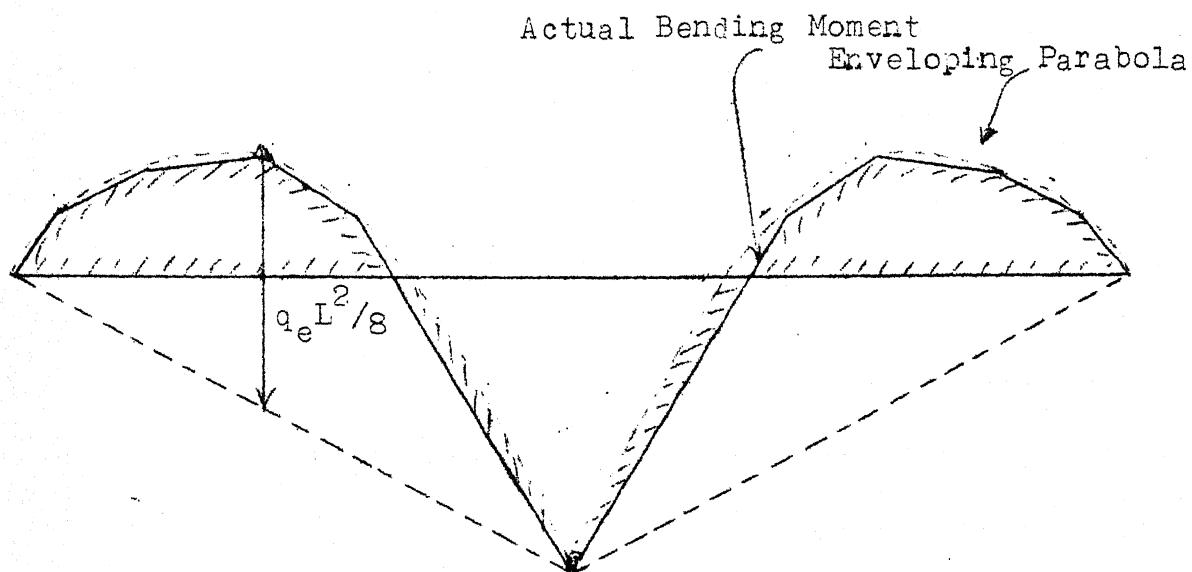
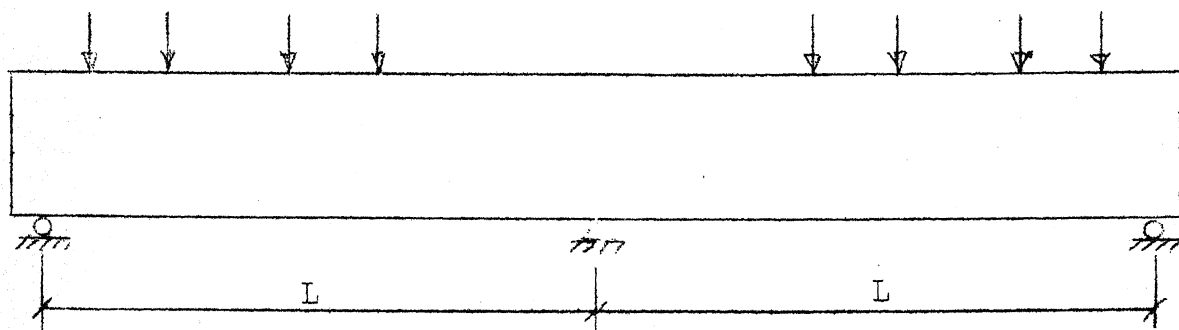


Fig. 2.47 Idealization of Live Loads For Qualitative Study

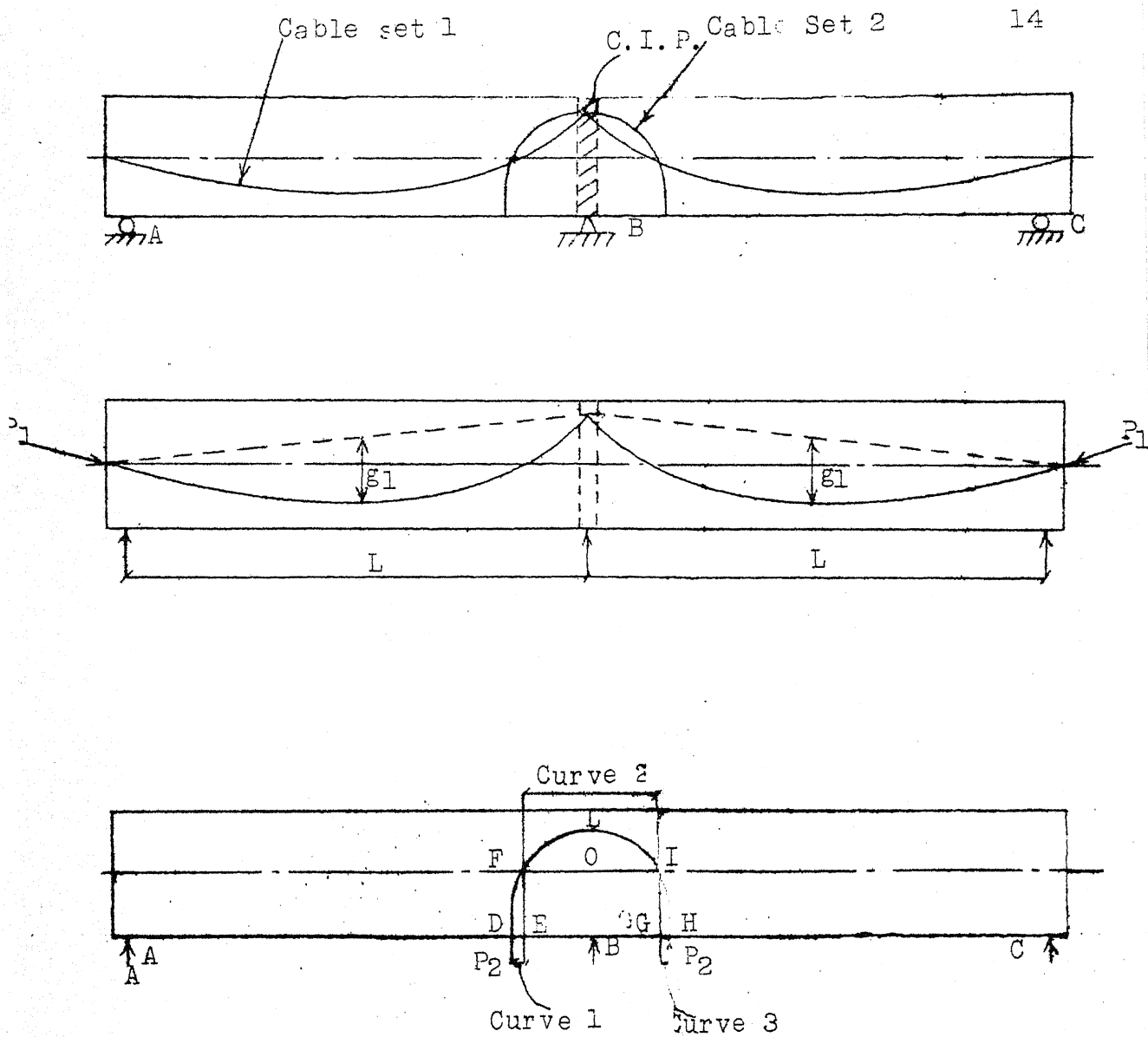
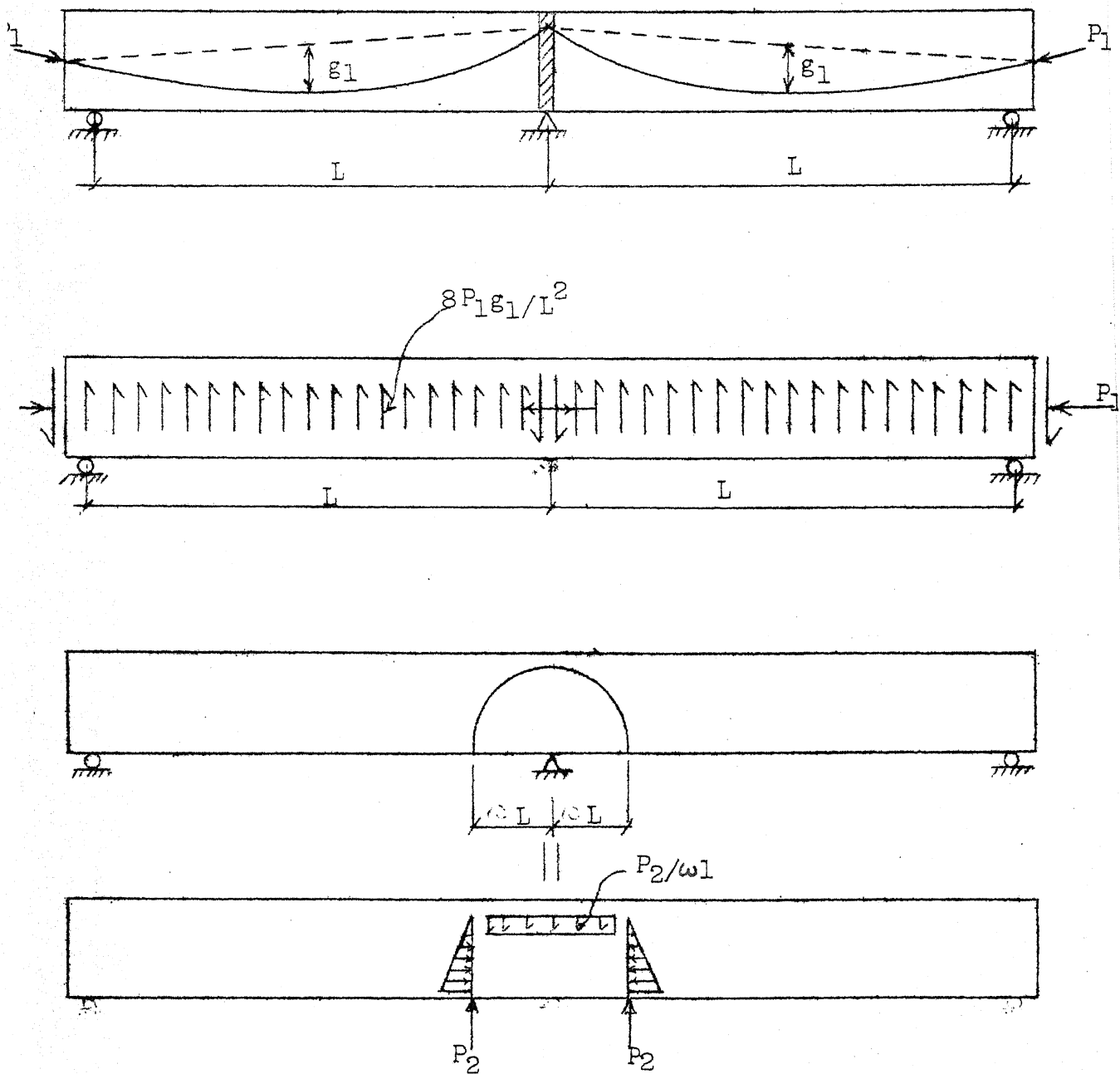
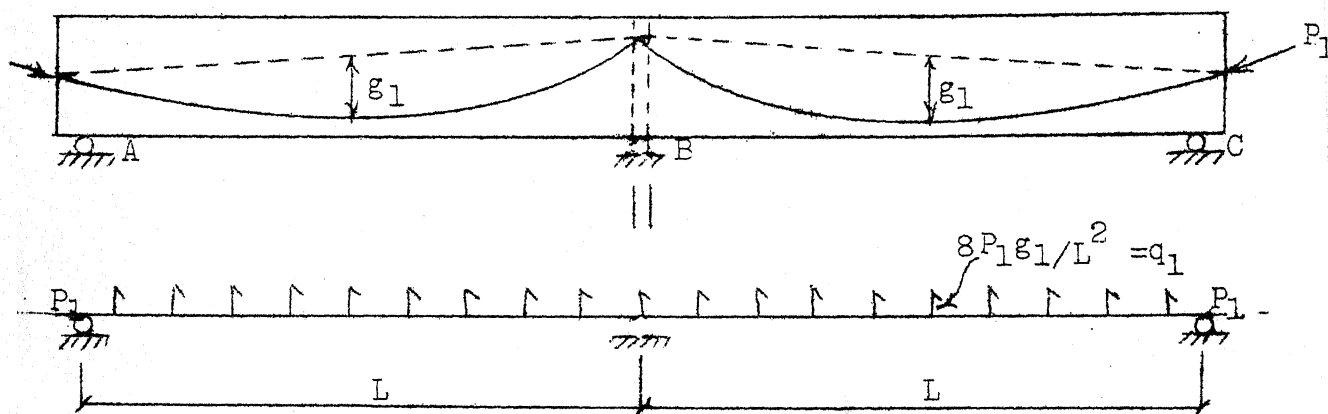


Fig.2.5 Cable Profiles

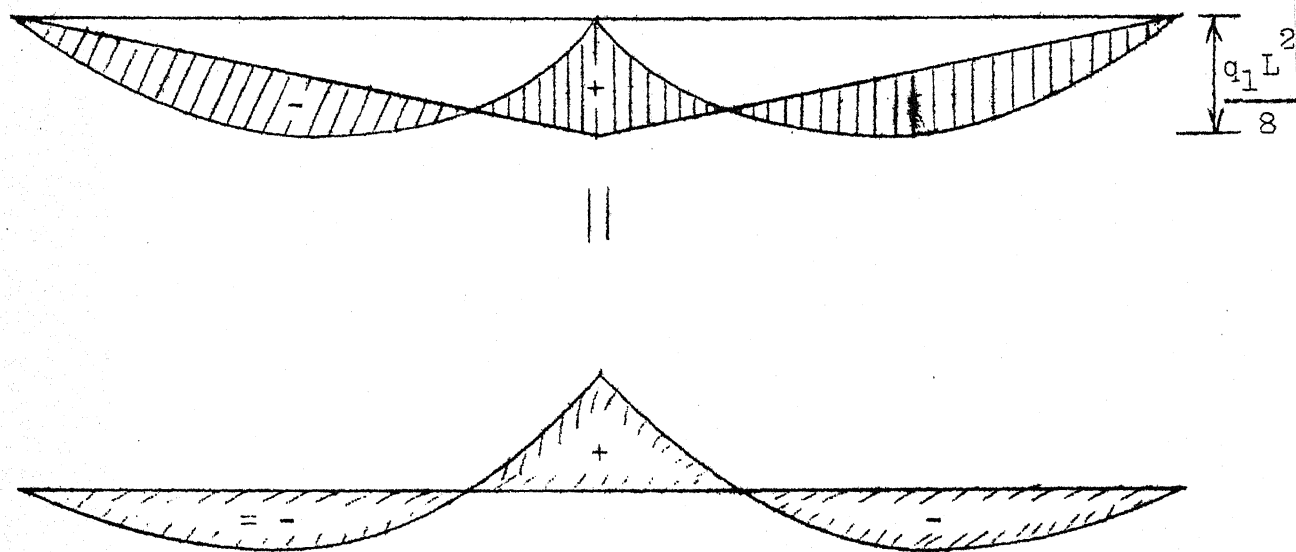


Equivalent Forces for Cable Set 2

Fig. 2.6 Equivalent Balancing Forces



(a) Balancing Forces Due To Cable Set 1



(b) B. M. D. Due To Cable Set 1

Fig.2.7 Balancing Force & B.M.D. Due to Cable Set 1

shown in Fig.2.6. The equivalent forces due to this cable also have been shown in Fig.2.6. The bending moment at any section due to parabolic cable profile can be deduced as (from Fig.2.7)

$$M_{xs} = \left[\frac{x}{L} \cdot \frac{1}{2} + \left(\frac{x}{L} \right)^2 \cdot \frac{1}{2} + \left(1 - \frac{x}{L} \right) 0.125 \right] q_1 L^2$$

where x is being measured from B (from mid support towards exterior support)

$$\text{Now } P_{1g_1} = q_1 L^2 / 8$$

$$\therefore q_1 L^2 = 8 P_{1g_1}$$

where g_1 = sag of the parabolic cable in single span

$$\therefore M_{xs} = \left[-X \cdot \frac{1}{2} + X^2 \cdot \frac{1}{2} + (1-X) 0.125 \right] 8 P_{1g_1} \quad (2.2)$$

where $X = x/L$

b) The Continuity Cable :

The continuity cable is provided as shown in the Figs.2.5 and 2.8. The continuity cable consists of three curves.

- i) Curve DF
- ii) Curve FI
- iii) Curve IH

Curve DF and Curve IH are assumed identical but of mirror reflection.

The slopes are matched at the common point of any two adjacent curves so that the two adjacent curves have smooth transition without any kink.

Equation of Curve 1 :

$$x_1 = c \left(\frac{h}{2} - y_1 \right) \left(\frac{h}{2} + y_1 \right) \quad (2.3)$$

satisfies the condition (at $y = h/2$, $x = 0$)

where c = arbitrary constant to be evaluated from the boundary

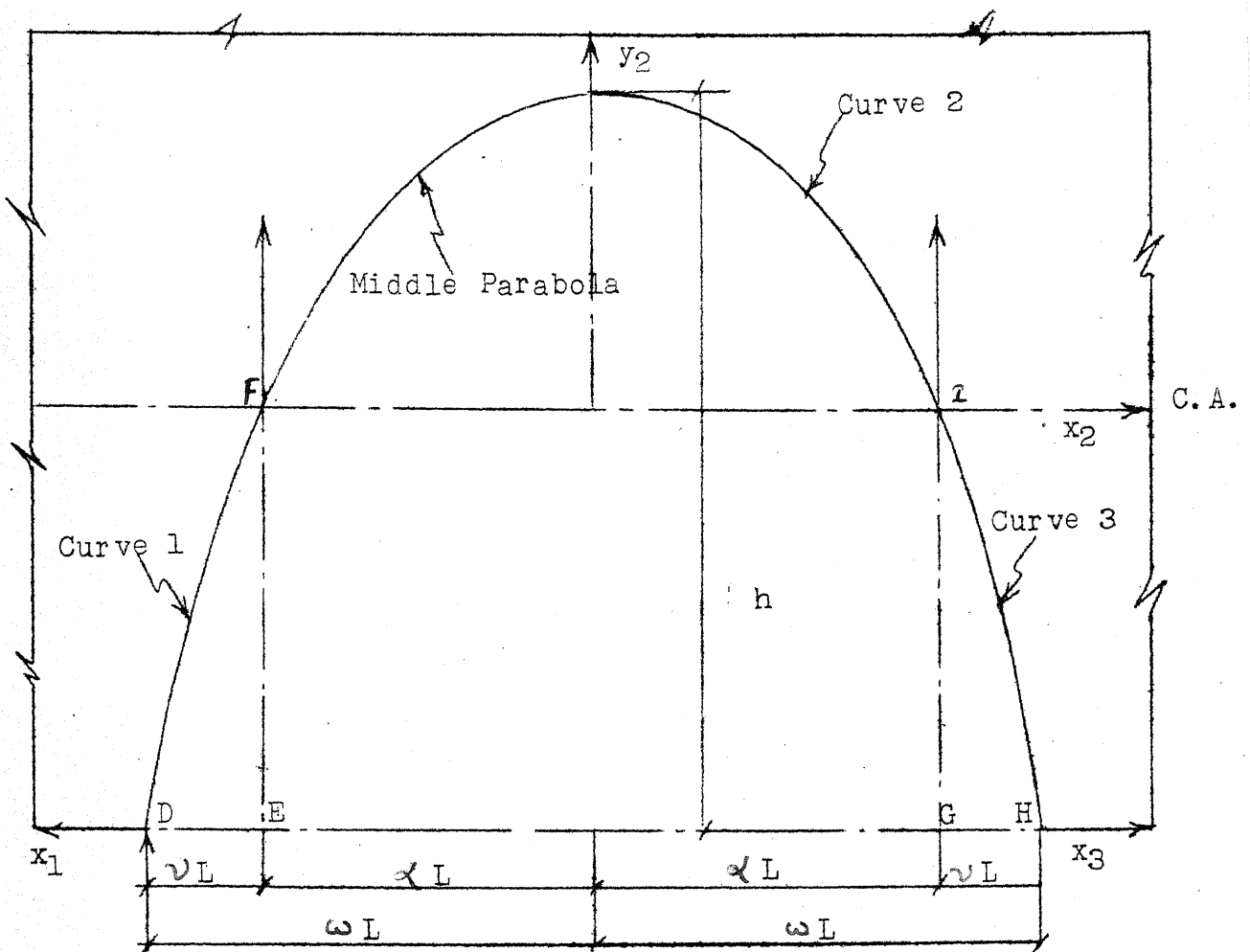


Fig.2.8 Continuity Cable

conditions.

h = depth of the beam

with E as origin,

$$\text{at } y = 0 \quad \frac{dx_1}{dy_1} = 0$$

$$\left. \frac{dx_1}{dy_1} \right|_{y=h/2} = -ch$$

Equation of the curve 2 :

$y = c_1 (\alpha L - x) (\alpha L + x)$ satisfies the condition :

(at $x = \alpha L$, $y = 0$)

where c_1 = arbitrary constant to be evaluated from the boundary condition.

L = length of the beam

αL = length of the curve 2 on either side of the support
B measured till the common point.

with O as origin

$$\text{At } x = 0, y = (\beta - 0.5) h = \gamma h$$

where

γh = Height of the curve 2 at support B measured from bottom fibre.

satisfying the above condition

$$\gamma h = c_1 \alpha^2 L^2$$

$$\therefore c_1 = \frac{\gamma h}{\alpha^2 L^2}$$

Equation of curve 2 :

$$y_2 = \frac{\gamma h}{\alpha^2 L^2} (\alpha L - x_2) (\alpha L + x_2) \quad (2.5)$$

Since a smooth transition between two adjacent curves is

required

$$\left[\frac{dy_2}{dx_2} \right]_{x=\alpha L} \cdot \left[\frac{dx_1}{dy_1} \right]_{y=h/2} = 1 \quad (2.6)$$

$$\therefore c = \alpha L/2 \gamma h^2$$

Equation of the curve 1 reduces to

$$x_1 = \frac{\alpha L}{2 \gamma h^2} \left(\frac{h}{2} - y_1 \right) \left(\frac{h}{2} + y_1 \right) \quad (2.7)$$

At $y_1 = 0$, $x_1 = \gamma L$

where $\gamma = \frac{1}{8} \cdot \frac{\alpha}{\gamma}$

Simplifying Eq. (2.7),

$$y_1^2 = \frac{h^2}{4} (1 - x_1/\gamma L) \quad (2.8)$$

Thus a continuity cable profile has been provided starting from a distance ωL on either side of the support and extending upto a height of βh

where $\omega = (\alpha + \gamma)$

$(\alpha + \gamma)$ has to be defined for optimal design.

The effect of the continuity cable would be three fold on the continuous beam (made continuous due to continuity cable profile).

The continuity cable has been replaced by the equivalent balancing forces (Fig.2.6). These equivalent balancing forces can be stated as :

i) A uniformly distributed load q_2 . q_2 is assumed to be distributed upto a length ωL since

$$\omega L = \alpha L + \gamma L$$

and $\gamma L \ll h/2$

ii) A concentrated prestressing force P_0 at the point

of anchorage.

Equating the sum of the vertical forces to zero gives

$$q_2 \omega L - P_2 = 0$$

$$\therefore q_2 = P_2 / \omega L \quad (2.9)$$

iii) A Distributed horizontal load up to a height of βh . A triangular distribution is assumed because of friction and curvature.

The balancing bending moment due to the above three factors has been calculated as follows :

1) Balancing bending moment due to q_2 :

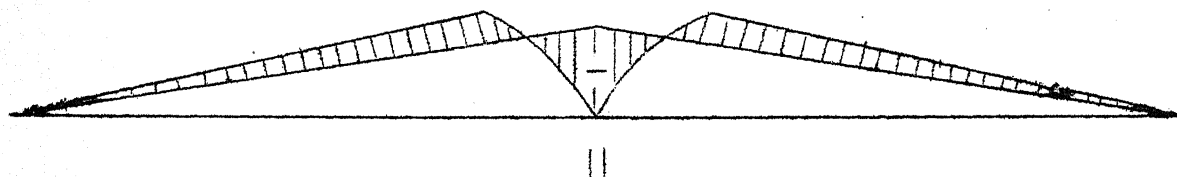
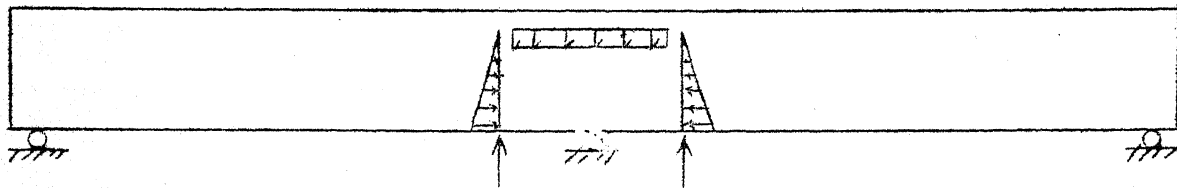
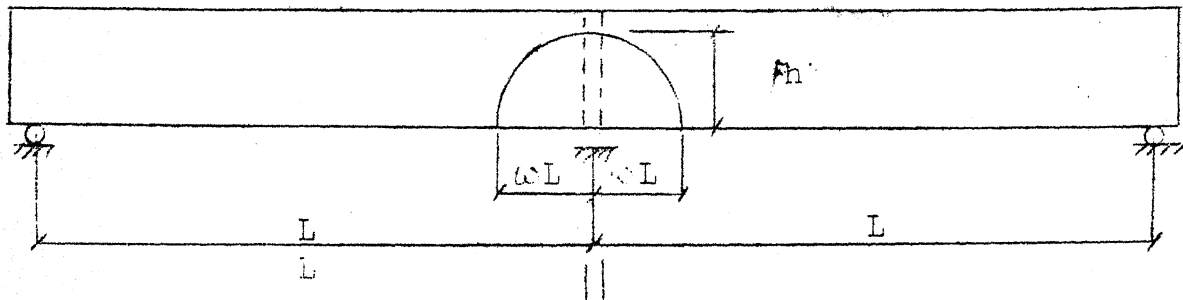
In Fig.2.9 the effect of q_2 has been taken into account. The moment distribution has been shown in Table 2.2 and the B.M. diag. has been shown in Fig.2.9.

The B.M. at any section due to continuity due to q_2 can be written as⁷

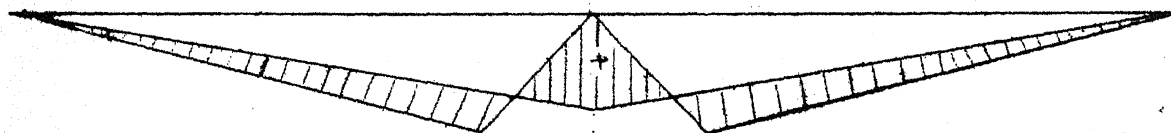
$$M_{xlc} = - (1-x/L) \left[\frac{q_2 \omega^3 L^2}{12} \left\{ 2 (3-4\omega) + 3\omega \right\} + \frac{q_2 \omega^3 L^2}{24} (4-3\omega) \right] + \left\{ \frac{q_2 \omega^3 L}{2} (L-x) \right\} - \left\langle \frac{q_2 (\omega L-x)^2}{2} \right\rangle$$

indicates that this magnitude is to be taken into account only if $(X - \omega) \leq 0$

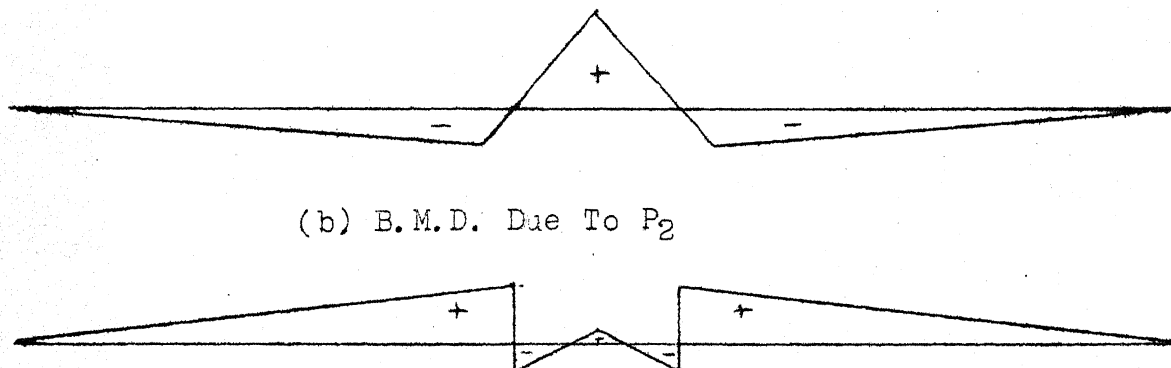
Moment Distribution for q_2 has been shown in Table 2.2 on the next page.



(a) B.M.D. Due To q_2



(b) B.M.D. Due To P_2



(c) B.M.D. Due To Eccentrically Acting Horizontal Force

Table 2.2 Moment Distribution for q_2

Joint	A		B		C
Member	AB	BA	BC	CB	
Relative stiffness	1.0	1.0	1.0	1.0	
D.F.	1.0	0.5	0.5	1.0	
Total	-	$M_{ba}^{q_2}$	$M_{bc}^{q_2}$	-	

$$M_{ba}^{q_2} = -\frac{q_2 \omega^2 L^2}{12} \left[2(3-4\omega) + 3\omega \right] - \frac{q_2 \omega^3 L^2}{24} (4-3\omega)$$

$$M_{bc}^{q_2} = \frac{q_2 \omega^2 L^2}{12} \left[2(3-4\omega) + 3\omega \right] + \frac{q_2 \omega^3 L^2}{24} (4-3\omega)$$

since, $q_2 = P_2/\omega L$

$$M_{xlc} = -(1-X) \left\{ \frac{P_2 L}{\omega} \left[\frac{\omega^2}{12} \left(2(3-4\omega) + 3\omega \right) + \frac{\omega^3}{24} (4-3\omega) \right] + \frac{P_2 L}{\omega} \left\{ \frac{\omega^2}{2} (1-X) - \left\langle \frac{1}{2} (\omega-X)^2 \right\rangle \right\} \right\} \text{ where } X = x/L \quad (2.10)$$

ii) Balancing B.M. due to concentrated force at anchorage :

In Fig.2.9 the effect of the concentrated force has been taken into account. The moment distribution has been shown in Table 2.3 and the bending moment diagram has been shown in Fig.2.9.

The B.M. at any section due to prestressing force at the anchorage can be written as⁷

$$M_{xlc} = (1-X) P_2 L \left[\omega(1-\omega)^2 + \frac{1}{2} \omega^2 (1-\omega) \right] + P_2 L \left[-\omega(1-X) + \left\langle \omega-X \right\rangle \right] \quad (2.11)$$

$\langle \rightarrow \rightarrow \rangle$ indicates that this magnitude is to be taken into account

only if $(X-\omega) \leq 0$

iii) B.M. due to the eccentrically acting horizontal force:

Effect of horizontal force has been shown in Fig. 2.6. The triangularly distributed load is replaced by a moment and axial force effects.

Since $L \ll h/2$

Table 2.3 Moment Distribution for the concentrated Prestressing force at Anchorage.

Joint	A		B		C	
Member	AB	BA	BC	CB	BC	CB
Relative stiffness	1.0	1.0	1.0	1.0	1.0	1.0
D.F.	1.0	0.5	0.5	1.0	0.5	1.0
Total	-	M_{ba}^{P2}	M_{bc}^{P2}	-	M_{bc}^{P2}	-

$$M_{ba}^{P2} = P_2 \omega L (1-\omega)^2 + \frac{1}{2} P_2 \omega^2 L (1-\omega)$$

$$M_{bc}^{P2} = - P_2 \omega L (1-\omega)^2 - \frac{1}{2} P_2 \omega^2 L (1-\omega)$$

this effect has been taken as laterally acting distributed load upto a height βh . The net force can be taken to be ηP_2 acting at an eccentricity of $(\frac{h}{2} - \frac{\beta h}{3})$ w.r.t. the centroidal axis where

$$\eta = \text{coefficient of losses due to prestressing}$$

$$M = \eta P_2 \left(\frac{h}{2} - \frac{\beta h}{3} \right) \quad (2.12)$$

acting at a distance $2\omega L/3$ from the middle support as shown

in Fig.2.9.

Table 2.4 Moment Distribution for eccentrically acting horizontal force.

Joint	A		B		C	
Member	AB	BA	BC	CB	CB	CB
Relative stiffness	1.0	1.0	1.0	1.0	1.0	1.0
D.F.	1.0	0.5	0.5	1.0	1.0	1.0
Total	-	$M_{ba}^{P_2}$	$M_{bc}^{P_2}$	-	-	-

$$M_{ba}^{P_2} = \eta P_2 h \left(.5 - \frac{B}{3} \right) (1-\omega) (1-3\omega) - \frac{1}{2} \eta P_2 h \left(.5 - \frac{B}{3} \right) \omega (2-3\omega)$$

$$M_{bc}^{P_2} = - \left[\eta P_2 h \left(.5 - \frac{B}{3} \right) (1-\omega) (1-3\omega) - \frac{1}{2} \eta P_2 h \left(.5 - \frac{B}{3} \right) \omega (2-3\omega) \right]$$

The B.M. at any section due to continuity cable due to M can be written as⁷,

$$M_{x3c} = \eta P_2 \left(\frac{h}{2} - \frac{Bh}{3} \right) \left[(1-\omega)(1-3\omega) - \frac{1}{2} \omega (2-3\omega) \right] (1-X) + \left[(1-X) - \langle 1 \rangle \right] \eta P_2 \left(\frac{h}{2} - \frac{Bh}{3} \right) \quad (2.13)$$

where $X = x/L$

$\langle - \rangle$ to be taken into account only if $X \leq 0$. The net B.M. at any section due to continuity cable is

$$M_{xc} = M_{x1c} + M_{x2c} + M_{x3c} \quad (2.14)$$

$$M_{xc} = - (1-X) (P_2 L / \omega) \left[\frac{\omega^2}{4} (2(3-4\omega) + 3\omega) + \frac{\omega^3}{24} (4-3\omega) \right] + (P_2 L / \omega) \left\{ \frac{\omega^2}{2} (1-X) - \left\langle \frac{1}{2} (\omega-X)^2 \right\rangle \right\} + (1-X) P_2 L \left[\omega (1-\omega)^2 + \frac{1}{6} \omega^3 (1-\omega) \right] + P_2 L$$

$$\left[-\omega(1-X) + \left(\frac{h}{2} - \frac{\beta h}{3} \right) \left[(1-\omega)(1-3\omega) - \frac{1}{2}(2-3\omega) \right] (1-X) + \right. \\ \left. \eta P_2 \left(\frac{h}{2} - \frac{\beta h}{3} \right) (1-X) - \langle 1 \rangle \right] \quad (2.15)$$

2.4 Continuity Compatibility :

A slope compatibility at the joint has to be established. The single spans will have different slope due to the first set of cables before the continuity cable is put in. The continuity cable must adjust the slopes to have a compatible continuous beam (Fig.2.10).

This is done by means of slope continuity. The beam in single span is taken and the slope at the middle support is calculated due to the different effects of parabolic cable and continuity cable and is matched at the middle support thus taking the continuity effect.

A) Slope due to the parabolic cable :

i) Slope due to q_1 :

This can be written as (Fig.2.11)

$$\theta_{11} = P_1 L_1 g_1 / 3 EI \quad (+ve) \quad (2.16)$$

ii) Slope due to horizontal eccentric force (Fig.2.11)

This can be written as,

$$\theta_{12} = \frac{P_1 e_1 L}{3 EI} \quad (-ve) \quad (2.17)$$

where E = Youngs Modulus of the section of the beam.

I = Moment of inertia of the section of the beam

B) Slope due to the continuity cable :

1) Slope due to q_2 :

This can be written as⁷, (Fig.2.9)

$$\theta_{21} = -\frac{q_2 \omega L^3 (2-\omega)^2}{24 EI} (-ve) \quad (2.18)$$

ii) Slope due to P_2 :

This can be written as⁷ (Fig.2.9)

$$\theta_{22} = \frac{P_2 \omega (1-\omega) (2-\omega) L^2}{6 EI} (+ve) \quad (2.19)$$

iii) Slope due to the moment $\eta P_2 (.5 - \beta/3) h$

This can be written as⁷ (Fig.2.9)

$$\theta_{23} = \frac{\eta P_2 (.5 - \beta/3) h}{6 EI} [3(1-\omega)^2 - 1] (-ve) \quad (2.20)$$

where, $\omega = (2/3) \omega$

iv) Slope due to the force ηP_2

This can be written as⁷ (Figs 2.9 & 2.11)

$$\theta_{24} = \eta P_2 (\beta - 0.5) h L / 3 EI \quad (-ve) \quad (2.21)$$

where $\eta = e^{-\mu \pi/2}$

μ = coefficient of friction losses

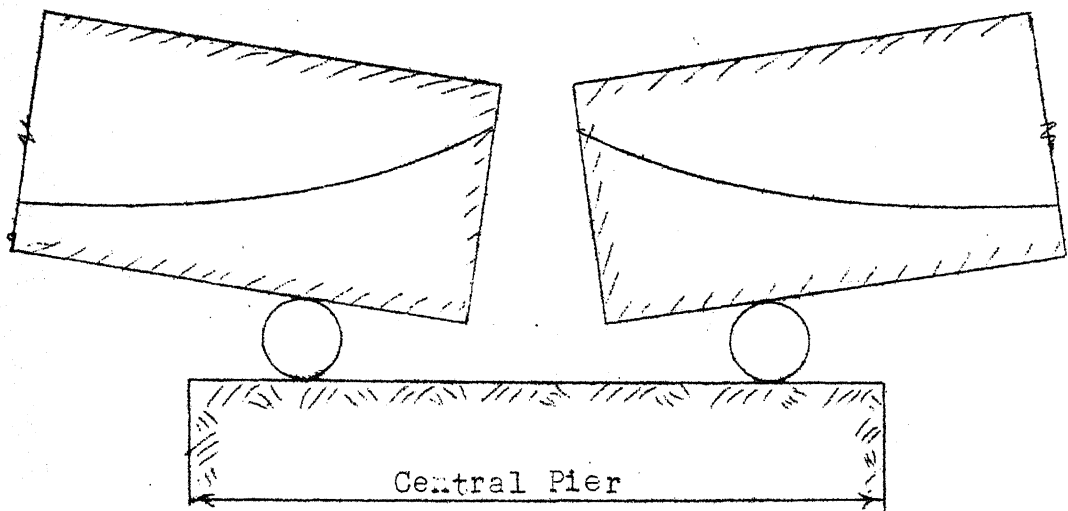
For continuity compatibility

$$\sum_{i=1}^2 \theta_{1i} + \sum_{i=1}^4 \theta_{2i} = 0 \quad (2.22)$$

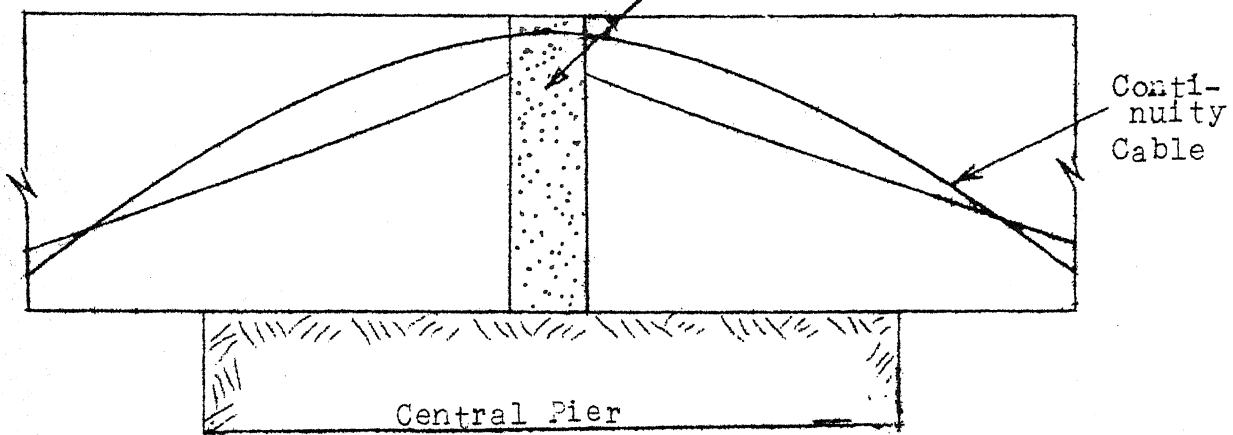
$$\left[\frac{P_1 L_1 g_1}{3 EI} - \frac{P_1 e_1 L}{3 EI} \right] + \left[-\frac{q_2 \omega L^3}{24 EI} (2-\omega)^2 \right] + \frac{P_2 \omega (1-\omega) (2-\omega) L^2}{6 EI} - \frac{\eta P_2 (.5 - \beta/3) h}{6 EI} [3(1-\omega)^2 - 1] - \frac{\eta P_2 (\beta - 0.5) h L}{3 EI} = 0 \quad (2.23)$$

Simplifying Eq.(2.23)

$$\frac{P_1 L}{3} (g_1 - e_1) = P_2 \left[\frac{\omega L^2 (2-\omega)^2}{24} - \frac{\omega (1-\omega) (2-\omega) L^2}{6} + \frac{\eta P_2 (.5 - \beta/3) h}{6} \right. \\ \left. [3(1-\omega)^2 - 1] + \frac{\eta P_2 (\beta - 0.5) h L}{3} \right] \quad (2.24)$$

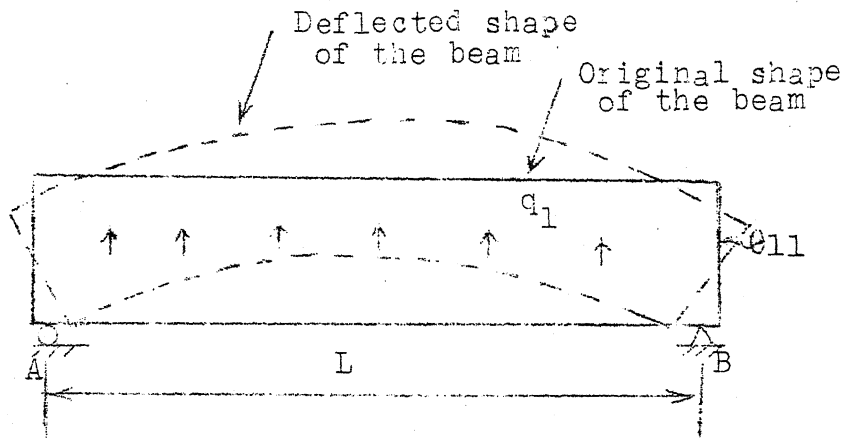
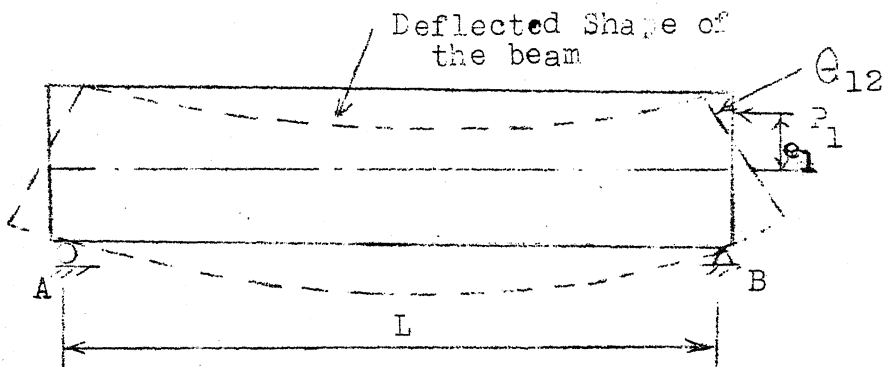


(a) Precast Beams Placed in Position
C.I.P.

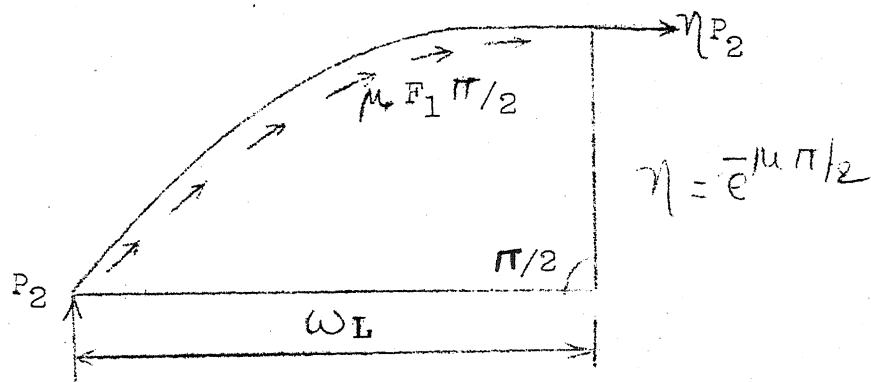


(b) Continuity Cable Anchored With C.I.P. Joint

Fig.2.10 Slope Matching Due to Continuity Cable

(a) Slope due to q_1 

(b) Slope due to eccentric force



(c) Losses Due to Prestressing

Fig.2.11 Slope due to the parabolic Cable and the effect of loss due to

Thus a relation between the two prestressing forces P_1 and P_2 has been established.

$$M_{xs} = (-X \cdot \frac{1}{2} + X^2 \cdot \frac{1}{2} + (1-X)0.125)8(r_1 P_2)g_1 \quad (2.25)$$

where $r_1 = P_1 / P_2$

The total bending moment at any section ($M_{xc} + M_{xs}$) from Eqs. (2.15) and (2.25) has to be balanced by the total dead and live load put together. Three design load conditions have been taken into consideration. The external bending moment in all the three cases would be different.

Case 1 External loading :

The net bending moment at any section x (Fig.2.1) can be written as

$$M_{xe1} = \left[\frac{1}{2}X - \frac{1}{2}X^2 - (1-X)0.125 \right] q_g L^2$$

Case 2 External loading:

The bending moment at any section (Fig.2.2) for span BA can be written as :

$$M_{xe2_1} = \left[\frac{1}{2}X - \frac{1}{2}X^2 - (1-X)(.125 + .063 q_1/q_g) \right] q_g L^2 \quad (2.27)$$

$$M_{xe2_2} = \left[\frac{1}{2}X - \frac{1}{2}X^2 \right] (1 + q_1/q_g) - (1-X) (.063 q_1/q_g + .125) q_g L^2$$

For Span BC

Case 3 External loading :

The bending moment at any section x (Fig.2.3) can be written as :

$$M_{xe3} = M_{xe1} (1 + q_1/q_g) \quad (2.28)$$

4th load condition :

In this case only q_g is acting. B.M. at any section x can be written as :

$$M_{xes} = \left[\frac{1}{2}X - \frac{1}{2}X^2 \right] q_g L^2 \quad (2.29)$$

CHAPTER III

DESIGN OF PRESTRESSING CABLE

3.1 Design Bending Moments :

The continuous beam must be designed for the effective bending moments due to the prestressing force and the external load. The bending moments produced by various effects must be superimposed for each load condition. The effect of the prestressing cable is to counterbalance the external force system. Balancing of the external load condition in a fixed load condition is very simple while it is not that easy in a multiple load condition. Since it is not possible to balance the external bending moment of various load conditions an optimization is very desirable. A precast construction is subjected to 2 sets of load conditions. The primary load conditions are those for which the structure should be designed. These load conditions are independent of the method of construction. The second set of load conditions are those associated with constructional method. Such loads are equally dominant in precast construction while these loads in cast in situ are marginal. The precast construction method in providing a continuity cable in continuous beams has a special advantage of governing loads. The handling and transportation loads of single spans are not of high dominance and therefore the continuous beam is designed for the primary loads and check is made on the handling conditions. The unbalance bending moment in each load condition may be expressed from previous chapter as :

$$M_{xubl} = M_{xe} + M_{xc} + M_{xs} \quad (3.1)$$

$$M_{xub2_1} = M_{xe1_1} + M_{xc} + M_{xs} \text{ for span AB} \quad (3.2)$$

$$M_{xub2_2} = M_{xe1_2} + M_{xc} + M_{xs} \text{ for span BC}$$

$$M_{xub3} = M_{xe3} + M_{xc} + M_{xs} \quad (3.3)$$

3.2 Minimization of unbalance Bending Moment :

The optimization is aimed at

$$\int_{-L}^L M_{xubl}^2 dx = \text{Minimum} \quad \text{for } l=1,2,3 \quad (3.4)$$

where,

M_{xubl} = unbalance bending moment for l^{th} load condition at distance x from the mid support.

It is not enough that the square of the total unbalance bending moment is minimum but is very essential that the maximum peak unbalance at any one point (either maximum B.M. or maximum -ve B.M.) should also be kept minimum. This condition is illustrated in Fig 3.1. It was found that the peak unbalance in the 2nd load condition (unsymmetric load) was critical. It is rather easy to select a cable profile for symmetric load conditions while the unsymmetric load condition (which is reversed in the bridges depending upon the movement of the loads) is found to be a critical criteria.

A simple computer programme was developed to minimize the unbalance bending moment. An approximate numerical integration procedure has been adopted as follows:

$$\bar{M}_{ubl} = (M_{ubl}^1)^2 = \sum_{i=1}^n (C_{p1}^i P_2 h + C_{g1}^i g L^2)^2 \quad (3.5)$$

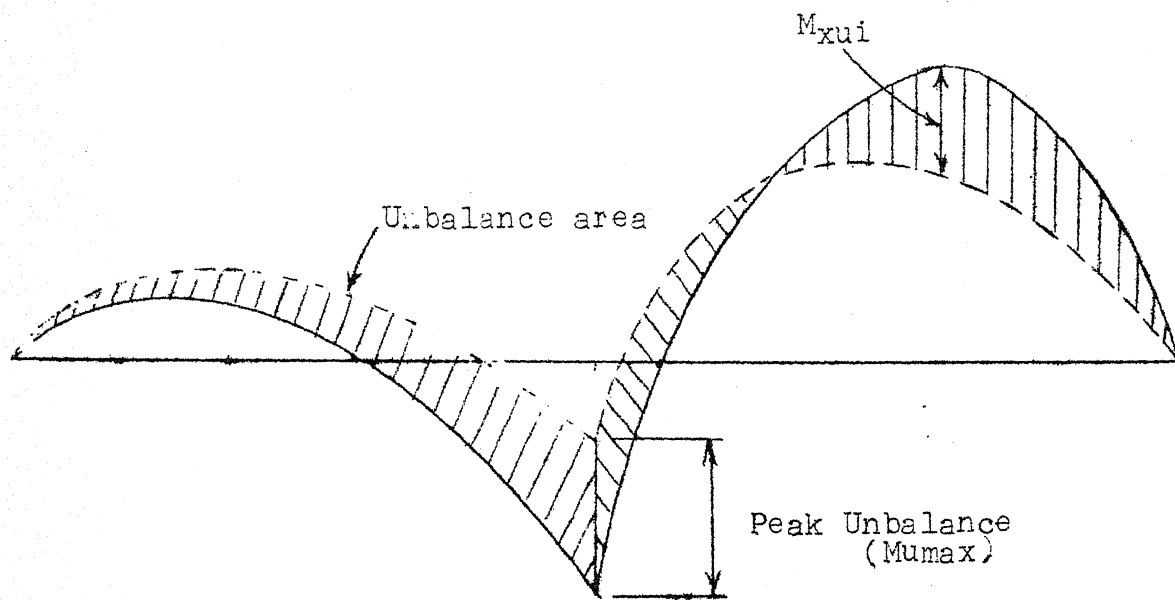


Fig. 3.1 Typical Unbalance B.M.D. For a Load Condition

where,

C_{p1}^i = Coefficient of the bending moment due to prestressing force for 1th load condition at ith point.

C_{g1}^i = Coefficient of the B.M. due to external load (expressed in terms of $q_g L^2$) at ith point for 1th load condition.

Minimization is affected by :

$$\frac{\partial \bar{M}_{ubl}}{\partial P_2} = 0 \quad \text{for } i = 1, 2, 3 \quad (3.6)$$

For case 1 loading :

$$2P_2 h \left[\sum_{i=1}^n (C_{p1}^i)^2 \right] = -2q_g L^2 \sum_{i=1}^n C_{g1}^i C_{p1}^i \quad (3.7)$$

For case 2 loading :

Span BA :

$$P_2 h \left[\sum_{i=1}^n (C_{p2-1}^i)^2 \right] = -q_g L^2 \sum_{i=1}^n C_{g2-1}^i C_{p2-1}^i$$

$$P_2 h \left[\sum_{i=1}^n (C_{p2-2}^i)^2 \right] = q_g L^2 \sum_{i=1}^n C_{g2-2}^i C_{p2-2}^i \quad (3.8)$$

--- For span BC

For case 3 loading

$$2P_2 h \left[\sum_{i=1}^n (C_{p3}^i)^2 \right] = -2q_g L^2 \sum_{i=1}^n C_{g3}^i C_{p3}^i \quad (3.9)$$

for both the spans

Adding Eqs. (3.7) to (3.9) and simplifying

$$P_2 h = -q_g L^2 \left[\frac{2 \sum_{i=1}^n C_{g1}^i C_{p1}^i + 2 \sum_{i=1}^n C_{g2-1}^i C_{p2-1}^i + 2 \sum_{i=1}^n C_{g2-2}^i C_{p2-2}^i + 2 \sum_{i=1}^n C_{g3}^i C_{p3}^i}{2 \left[\sum_{i=1}^n (C_{p1}^i)^2 + \sum_{i=1}^n (C_{p3}^i)^2 \right] + \sum_{i=1}^n (C_{p2-1}^i)^2 + \sum_{i=1}^n (C_{p2-2}^i)^2} \right] \quad (3.10)$$

The variable in the minimization programme is P_2 which is nondimensionalized as,

$$\lambda = \frac{P_2}{q_g L^2} \quad (3.11)$$

$$\lambda = - \frac{2 \left[\sum_{i=1}^n c_{g1}^i c_{p1}^i + c_{g3}^i c_{p3}^i \right] + \left[\sum_{i=1}^n c_{g2_1}^i c_{p2_1}^i + \sum_{i=1}^n c_{g2_2}^i c_{p2_2}^i \right]}{2 \left[\sum_{i=1}^n (c_{p1}^i)^2 + \sum_{i=1}^n (c_{p3}^i)^2 \right] + \left[\sum_{i=1}^n (c_{p2_1}^i)^2 + \sum_{i=1}^n (c_{p2_2}^i)^2 \right]} \quad (3.12)$$

The values of P_2 & λ are evaluated from the above minimization with the help of computer. The flow chart for the computer program made is shown in Fig 3.2.

3.3 Normalization of Data :

There are few design data that have to be normalized in this procedure. The eccentricity of the cables is always chosen as maximum, since maximum eccentricity minimizes the area of steel and maximizes the ultimate moment capacity of the section. Therefore, the following quantities are selected:

gh = sag of the single span cable (maximum)

βh = eccentricity of the continuity cable at mid support
(maximum)

These are restricted by cover requirement and therefore they are selected in the range of

$$g = 0.55 \text{ to } 0.65 \quad (3.13)$$

$$\beta = 0.6 \text{ to } 0.9 \quad (3.14)$$

The minimization of the unbalance bending moment is possible only if the relative ratio of the bending moments due to self weight and live load are known. This ratio is fixed once the bridge is designed and varies with the span and type of the design load. Therefore, the ratio of the range is selected as :

$$M_1/M_g = r = 0.5 \text{ to } 3.0 \quad (3.15)$$

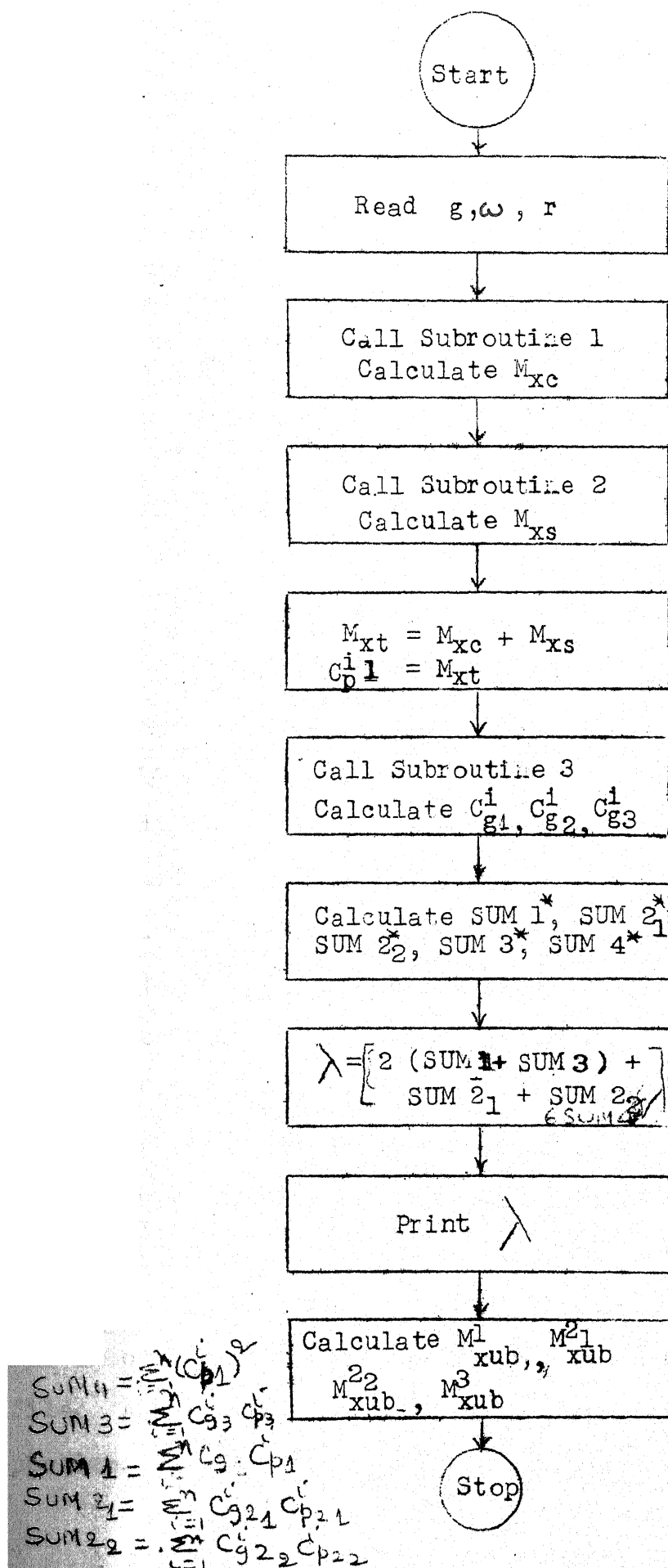
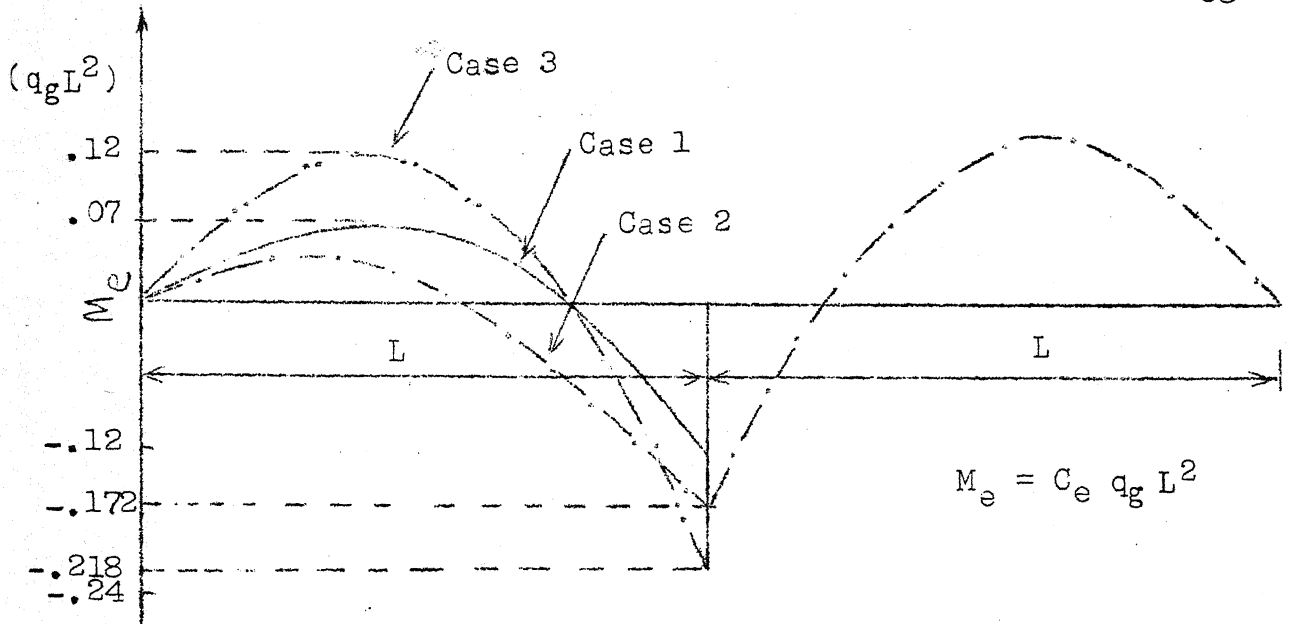


Fig.3.2 Flow Chart for the Computer programme used.

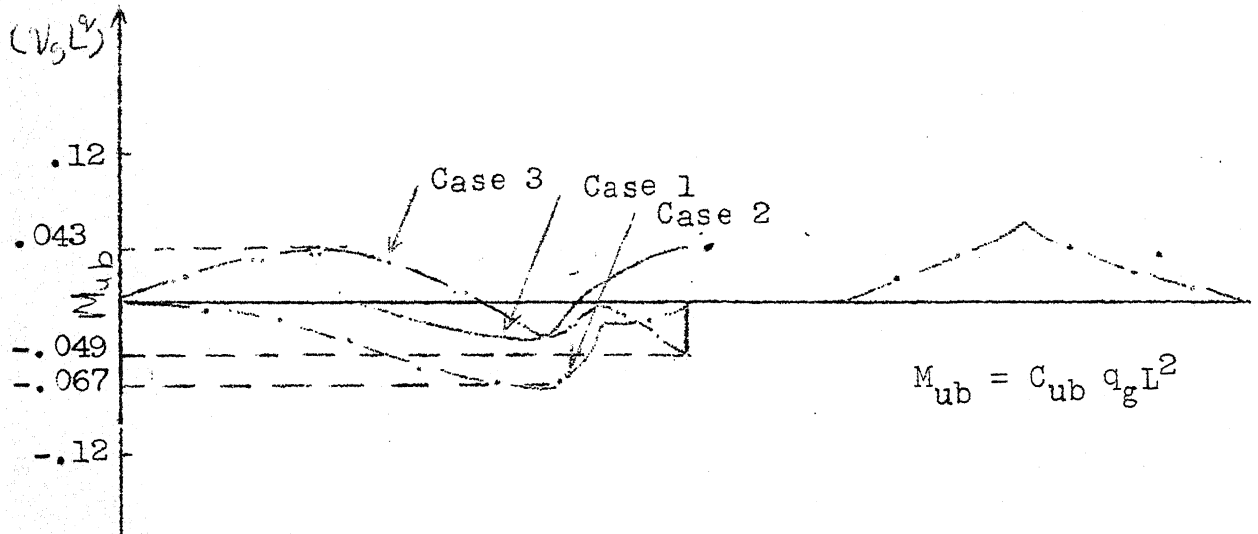
3.4 Results

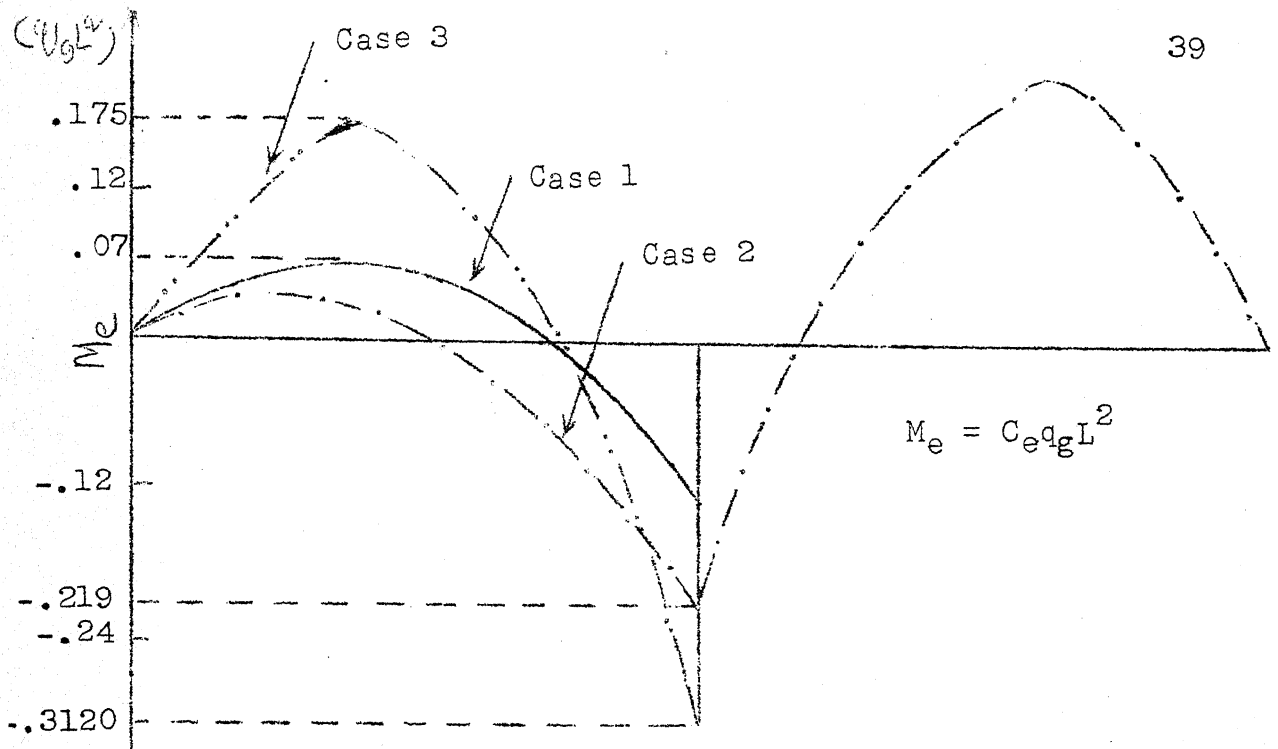
Typical unbalance (effective design) bending moment diagrams which are obtained after optimization are shown in Figs. 3.3 and 3.4. For a fixed parameters of r , g and ω it could be seen from Fig. 3.3 that the unbalance peak bending moment is $.067q_g L^2$ while the maximum magnitudes in the other two load conditions are in the order of $.049 q_g L^2$. The absolute maximum bending moment due to the external load occurs when the total span is subjected to the live load and the order of magnitude is of $0.218 q_g L^2$ occurring at the mid support. An appropriate continuity cable can reduce the maximum bending moment by about 75 percent.

The main object of the investigation is to provide some data regarding the design of prestressing force and the cable profiles. It is assumed that the total depth of the beam is arrived based on other design criteria such as architecture or deflections etc. The sag of the simply supported cable may be 0.55 to 0.65 depending on the cover limits. The relation between the two prestressing forces is given by Eq. (2.24). An approximate relation (r) between the live load and the self weight moments has to be assumed. The required prestressing force is then obtained from Figs. 3.12 and 3.13. The designer has an option to select the span of the continuity cable profile. An illustrative example is given for clarity of the problem.

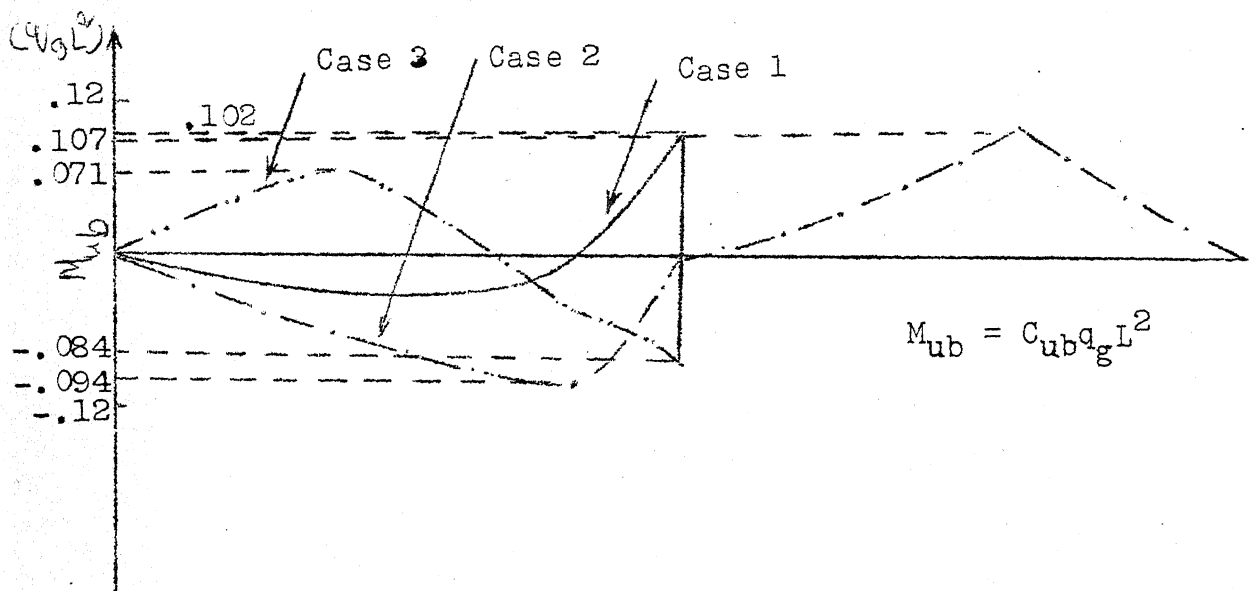


(a) External Bending Moment Diagram

(b) Unbalance Bending Moment Diagram
($g = .55, \omega = 0.25, r = 0.75$)Fig.3.3 Design Bending Moment Diagram
(for $g = 0.55$ & $\omega = 0.25$)



(a) External Bending Moment Diagram



(b) Unbalance Bending Moment Diagram
 $(g = 0.6, \zeta = 0.2, r = 1.5)$

3.5 Design Example

Design of a 100 m 2-span continuous prestressed concrete girder for class A loading. Assume post tensioned beams with M-450 concrete.

1) Design for Antisymmetric loading :

The beam with the particulars of the class A loading is shown in Fig.3.5. The idealization of the bending moment diagram due to the class A loading (drawn after moment distribution in Table 3.1) by an enveloping parabola is also shown in Fig.3.5. The idealized equivalent distributed load is not to be used in actual design process but is only a tool in the development of the design parameters .

From Fig. 3.5

$$\frac{q_1 L^2}{8} = 549,867 \text{ kg/m}$$

$$\therefore q_1 = 1759 \text{ kg/m}$$

$$\text{Let } r = 0.75$$

$$\therefore q_g = 2345 \text{ kg/m}$$

Selection of Prestress :

Choosing $\omega = 0.25$, $g = 0.55$ as shown in Fig.3.5

$$\lambda = 0.027 \text{ (from Fig.3.13)}$$

$$\therefore \lambda = 0.027 = \frac{P_2 e h}{q_g L^2}$$

Table 3.1 Moment Distribution for Class A loading
for antisymmetric case*

Joint	A		B		C
Member	AB	BA	BC	CB	
Relative stiffness	1.0	1.0	1.0	1.0	
D.F.	1.0	0.5	0.5	1.0	
F.e.m.	-	-	+ 355,507	-319,923	
Balance	-	-167,753.5	- 167,753.5	+319,923	
C.O.	-83,876.75	-	+ 159,901.5	-83,876.75	
Balance	+83,876.75	-79,980.75	- 79,980.75	+83,876.75	
C.O.	-39,990.37	+ 41,938.37	+ 41,938.37	-39,990.37	
Balance	+39,990.37	- 41,938.37	- 41,938.37	+39,990.37	
Total	-	-247,734.25	+ 247,734.25	-	

* Bending Moment is in Kg.m.

where

P_{2e} = Prestressing force at working condition in the cable

Set 2

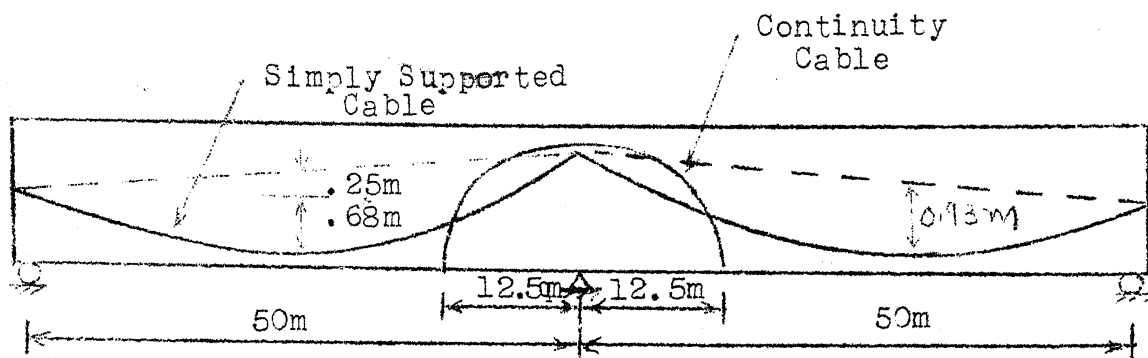
Choosing $h/L = 1/30$

$$\therefore P_{2e} = 94,986 \text{ kg.}$$

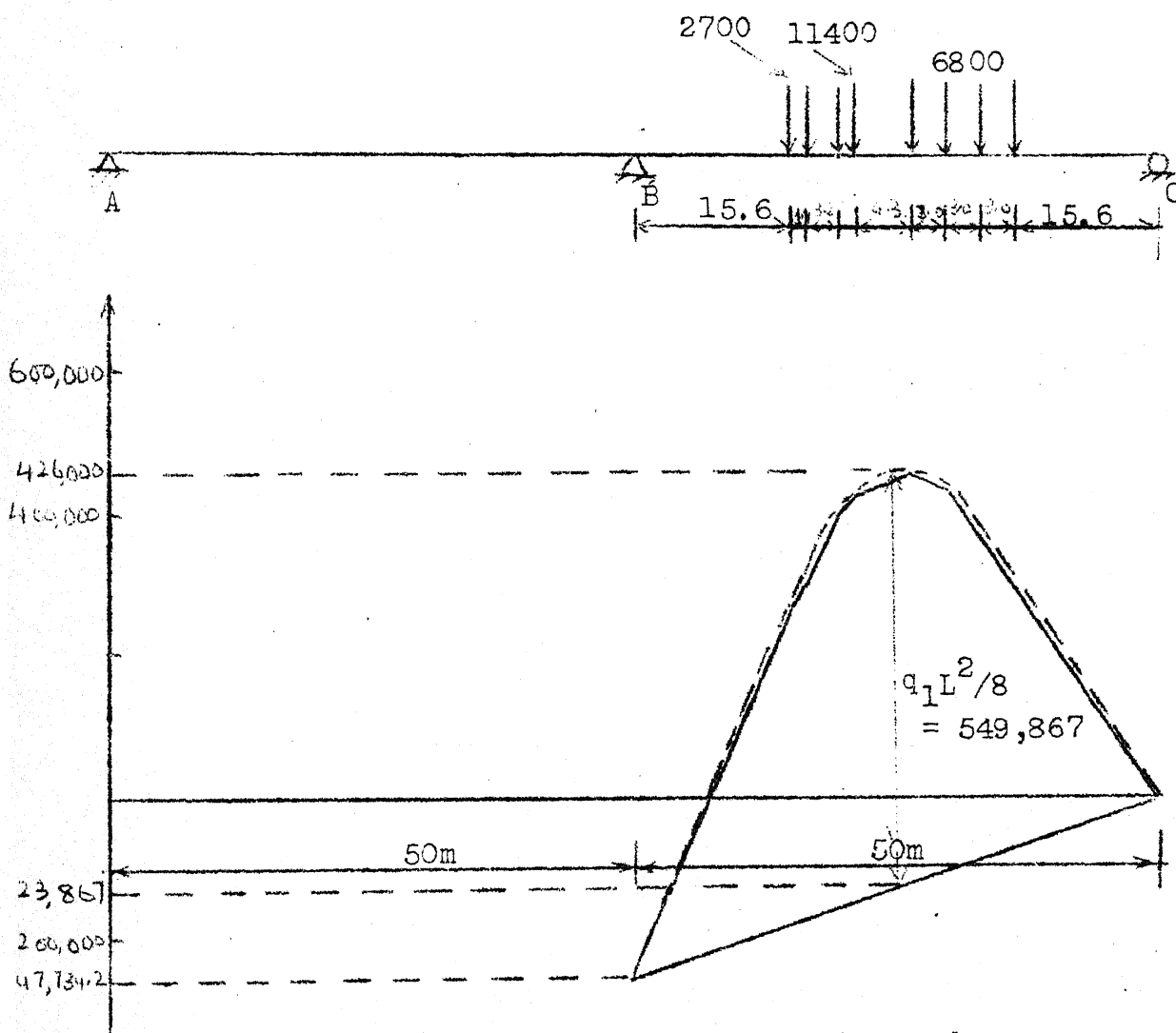
From Eq.(2.24)

$$P_{1e} = 7.3 P_{2e}$$

$$= 7.3 \times 94,986 = 693,398 \text{ kg.}$$



(a) Beam showing Cable Profiles



(b) Beam showing Class A Loading and the B.M. Diagram

Fig.3.5 Idealization of net Bending Moment Diagram by an equivalent u.d.l. for antisymmetric case

where

P_{1e} = Prestressing force at working condition in Cable Set 1

$$P_e = P_{1e} + 0.53 P_{2e} \text{ (at mid support)} \quad (3.16)$$

where

P_e = Total prestressing force at working at the middle support.

$$\begin{aligned} \therefore P_e &= 693,398 + 0.53 \times 94,986 \\ &= 743,741 \text{ kg (at mid support)} \end{aligned}$$

$$\text{Let } f_{av} = 2/3 \times 0.31 f'_c$$

where

$$\begin{aligned} f'_c &= \text{maximum permissible stress in compression} \\ &= 2/3 \times 0.31 \times 450 \text{ kg/cm}^2 \\ &= 100 \text{ kg/cm}^2 \end{aligned}$$

$$\therefore A = \frac{743.741}{100 \times 100} \text{ m}^2 = 0.74 \text{ m}^2$$

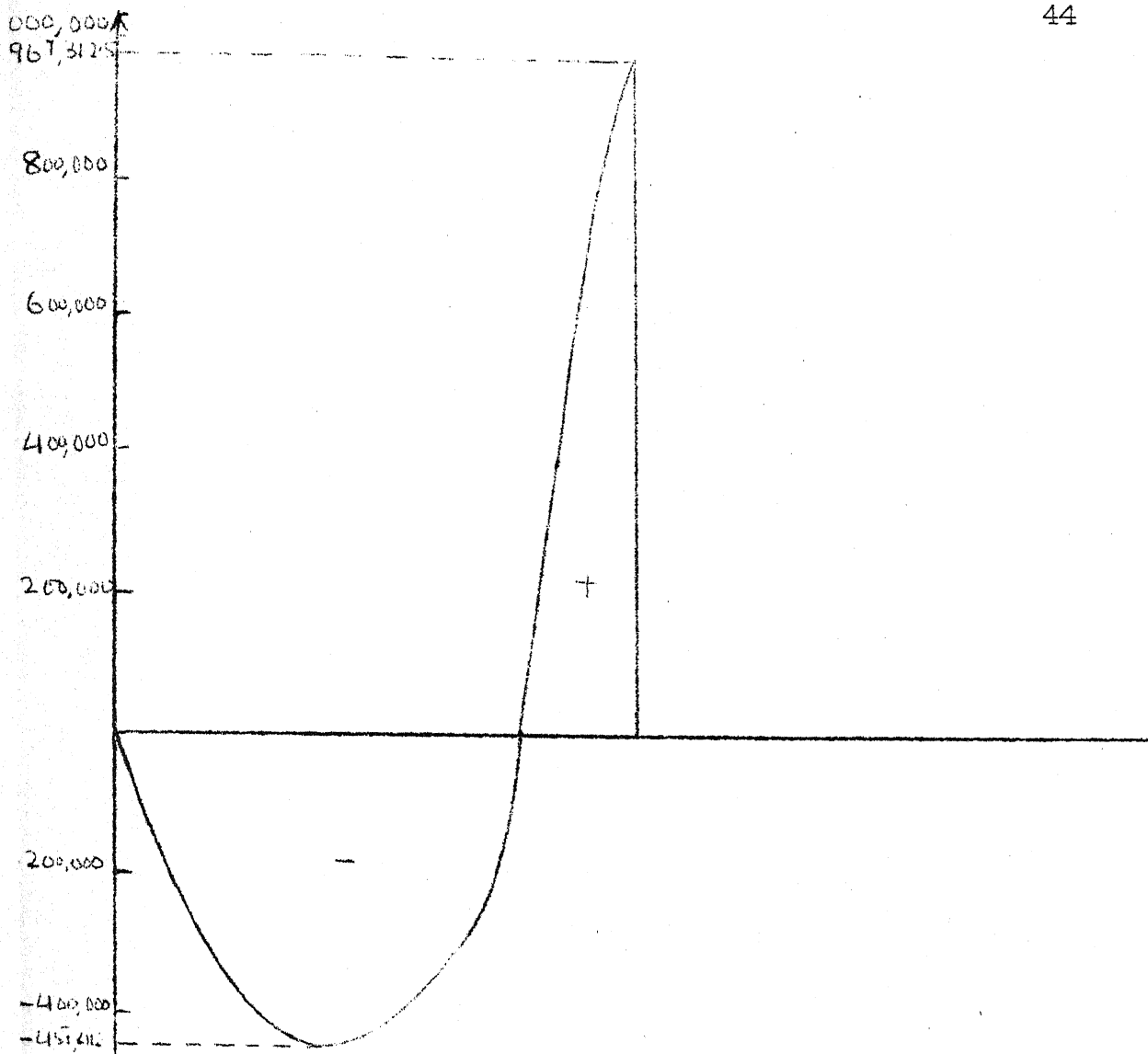
where A = cross section area

$$\text{Provide } A = 0.80 \text{ m}^2$$

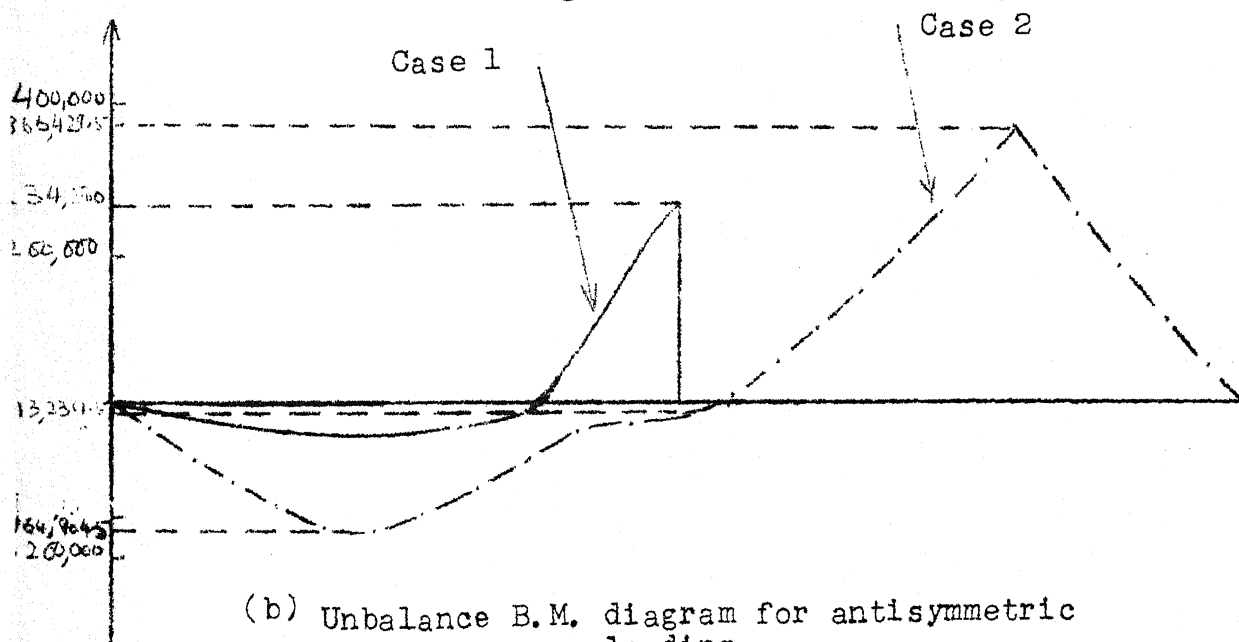
Checking of stresses :

For any IRC loading chosen there will be several load conditions depending upon the movement of the wheels. For each load condition there will be at least two critical section for positive bending moment and for negative bending moment. The beam designed has to be checked at these critical sections for both transfer and working conditions.

Transfer condition:



(a) Net B.M. diagram due to Prestressing forces



(b) Unbalance B.M. diagram for antisymmetric loading

Fig.3.6,

Maximum Design B.M. = + 234,500/0.85 at mid support
 The expression for stress at transfer in compression can be deduced as⁸ :

$$f_{ct} = \frac{1}{0.85} \frac{P_e}{A} + \frac{M_{bt}}{A} \left(\frac{\Delta}{1+\Delta} \right) \frac{1}{\rho} \cdot \frac{1}{h} \quad (3.17)$$

where

f_{ct} = stress at transfer in compression

M_{bt} = Design bending moment at transfer condition

ρ = efficiency ratio = $\frac{r_g^2}{h^2}$

Δ = Shape factor = y_b/y_t

$\frac{P_e}{0.85}$ = Transfer prestressing force

r_g = Radius of gyration of the section

y_b = Distance of the N.A. from the bottom flange

y_t = Distance of the N.A. from the top flange

Choosing the value of ρ and Δ as follows

$$\rho = 0.1529$$

$$\Delta = 1.0$$

A symmetric section has been chosen because the maximum + ve and maximum unbalance B.M. have been taken into consideration.

$$\begin{aligned} f_{ct} &= \frac{1}{0.85} \times \frac{743,741}{8,000} + \frac{275,882 \times 100}{8,000} \times (1/2) \times \frac{1}{0.1529} \times \frac{1}{170} \\ &= 109.3 + 67.6 \\ &= 176.9 \text{ kg/cm}^2 \quad (47 f_c) \end{aligned}$$

$$f_{tt} = -109.3 + 67.6$$

$$= -41.7 \text{ kg/cm}^2 \text{ (compression)}$$

where

f_{tt} = stress at transfer in tension

Check for simply supported case :

Beam with simply supported cable profile, has been taken into consideration in this case before the continuity cable is provided.

$$f_{ct} = \frac{1}{0.85} \frac{693,398}{8000} + \left[\frac{1}{0.85} \times 693,398 \times 51 \right] \frac{1}{8000} x^{(1/2)} x \frac{1}{0.1529} x \frac{1}{170}$$

$$= 101.97 + 101.9$$

$$= 203.87 \text{ kg/cm}^2 \quad (< 0.47f'_c)$$

$$f_{tt} = -101.97 + 101.9$$

$$= -.07 \text{ kg/cm}^2 \quad (\text{compression})$$

The stresses are within the allowable limit.

Working load condition :

From Fig.3.6 since working load condition is being considered for antisymmetric loading case 2 has to be taken into account.

Maximum Design B.M. = + 366,429.5kgm (at $x = 0.6L$ from mid support)

The expression for stress at working in compression can be deduced as⁸ :

$$f_{ce} = \frac{P_{1e}}{A} + \frac{M_{be}}{A} \left(\frac{\Delta}{1+\Delta} \right) \frac{1}{\rho} \cdot \frac{1}{h} \quad (3.18)$$

where

M_{be} = Design B.M. at working load condition (at $x=0.6L$ from the mid support)

$$f_{ce} = \frac{693,398}{8000} + \frac{366,429.5 \times 100}{8000} x^{(1/2)} \frac{1}{0.1529} x \frac{1}{170}$$

$$\begin{aligned}
 f_{ce} &= 86.67 + 89.8 \\
 &= 176.47 \text{ kg/cm}^2 (\approx .38f'_c) \text{ O.K.} \\
 f_{te} &= -86.67 + 89.8 \\
 &= +3.13 \text{ kg/cm}^2 (< .02f'_c) \text{ O.K.}
 \end{aligned}$$

The stresses are within the allowable limit.

For $\rho = 0.1529, \Delta = 1.0$

$$b_t = \frac{8000}{2600 \times 170} = 182 \text{ cm}$$

$$b_b = 182 \text{ cm}$$

$$t_t = .08 \times 170 \approx 14 \text{ cm}$$

$$t_b = 14 \text{ cm}$$

$$b_w = .12 \times 182 = 22 \text{ cm}$$

where

b_t = width of the top flange

b_b = width of the bottom flange

t_t = thickness of the top flange

t_b = thickness of the bottom flange

b_w = thickness of the web.

The section chosen is shown in Fig.3.8

2) Check for symmetric loading :

$$q_1 L^2 / 8 = 590, 786$$

$$\therefore q_1 = 1890 \text{ kg/m}$$

$$\therefore q_g = 2520 \text{ kg/m}$$

Cable profiles have been provided as shown in Fig.3.5

Check for stresses for symmetric load conditions :

$$\text{Maximum Design B.M.} = + \frac{252,000}{0.85} \text{ (at mid support)}$$

$$= + 296,470 \text{ kgm.}$$

$$f_{ct} = 109.3 + \left(\frac{296,470}{275,882} \right) \times 67.6$$

$$= 1866 \text{ kg/cm}^2 (< .47f'_c) \text{ O.K.}$$

$$f_{tt} = -109.3 + 72.3$$

$$= -37.0 \text{ kg/cm}^2 \text{ (compression) O.K.}$$

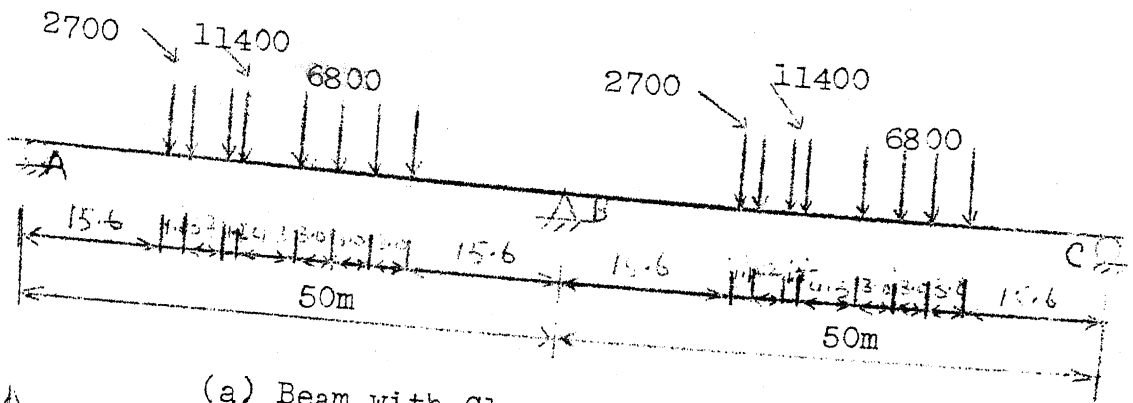
Working load condition :

From Fig.3.8(a) since working load condition is being considered for symmetric loading case 3 has to be taken into account.

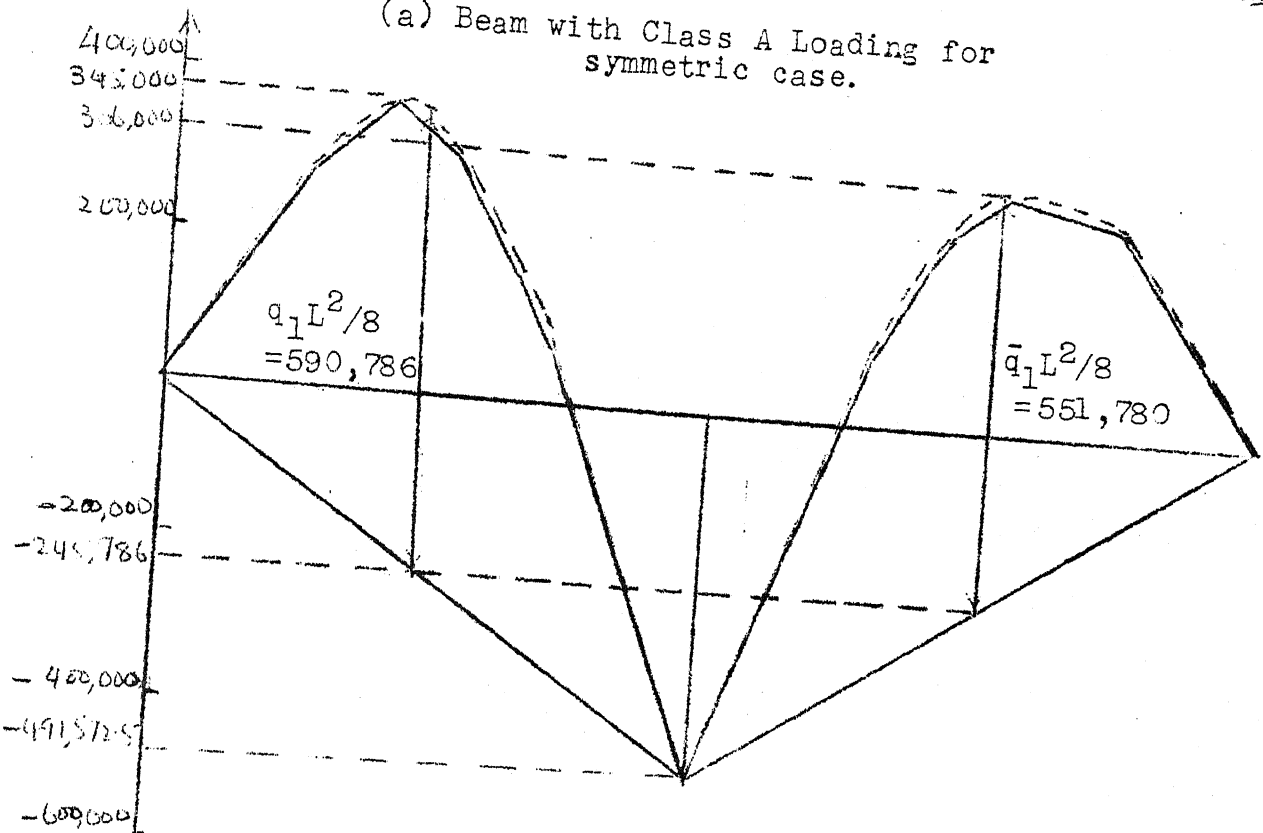
Table 3.2 Moment Distribution for class A loading for symmetric case*

Joint	A		B		C
Member	AB	BA	BC	CB	
Relative stiffness	1.0	1.0	1.0	1.0	
D.F.	1.0	0.5	0.5	1.0	
F.e.m.	+335,507	-319,923	+ 335,507	-319,923	
Balance	-335,507	- 7,792	- 7,792	+319,923	
C.O.	-3,896	-167,753.5	+ 159,961.5	- 3,896	
Balance	+3,896	+ 3,896	+ 3,896	+ 3,896	
C.O.	+1,948	+ 1,948	+ 1,948	+ 1,948	
Balance	-1,948	- 1,948	- 1,948	- 1,948	
Total	-	-491,572.5	+ 491,572.5	-	

* B. M. is in Kgm.

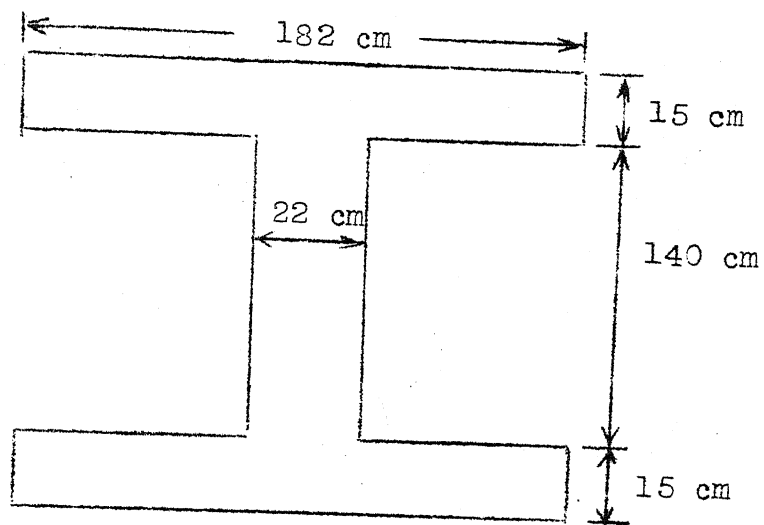
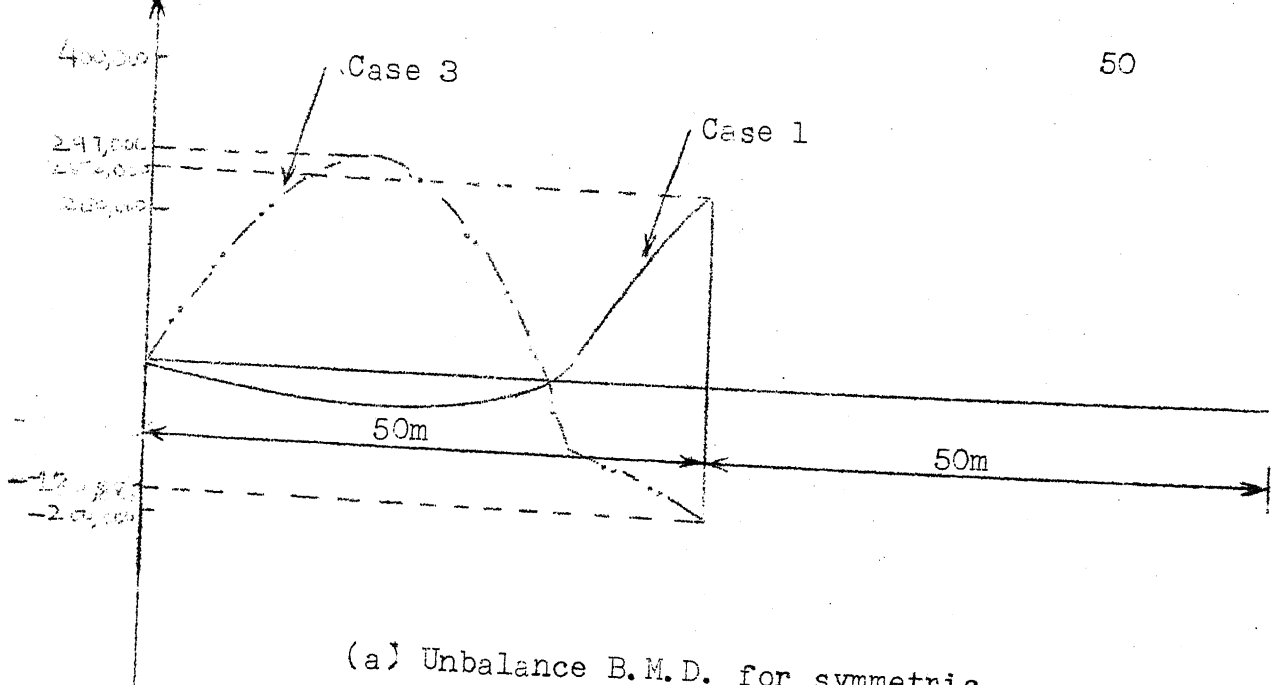


(a) Beam with Class A Loading for symmetric case.



(b) Bending Moment Diagram

Fig.3.7 Idealization of net B.M.D. by an equivalent u.d.l. for symmetric case



(b) Section of the Beam selected

Fig.3.8 Design B.M.D. and section of the beam

Maximum Design B.M. = + 297,900 kg.m. (at $x = 0.6L$ from mid support)

$$f_{ce} = 86.67 + \frac{297,900}{8000} \times 100 \times (1/2) \times \frac{1}{.1529} \times \frac{1}{170}$$

$$= 86.67 + 73.01$$

$$= 159.68 \text{ kg/cm}^2 \quad (< 0.38f'_c) \quad \text{O.K.}$$

$$f_{te} = - 86.67 + 73.01$$

$$= -13.66 \text{ kg/cm}^2 \quad (\text{compression}) \text{ O.K.}$$

Hence the stresses are within the allowable limit.

3.6 Discussion of Results

As discussed earlier there are many parameters that influence the design of continuity cable and simply supported cables. They are :

- 1) Maximum sag = gh
- 2) Span of the continuity cable = ωL
- 3) Maximum Eccentricity of the continuity cable = βh

This has been kept as constant (maximum)

The effect of the various parameters listed above on the unbalance bending moment has been studied and in the Figs. 3.9 and 3.10 these effects have been shown based on the results obtained by the computer programme.

3.6.1 Effect of Maximum sag

The designer has a choice in selecting :

- 1) The sag (gh) in the simply supported case
- 2) The span (ωL) of the continuity cable .

Higher the sag lower is the steel requirement provided the stresses at transfer are within the permissible limits. The

problem is worked with $g=0.55$ and 0.6 . These two values may be considered as the limiting sags subjected to the cover limits. It may be observed from Figs.3.9 and 3.10 that the unbalance bending moment is affected only marginally by the change in the value of the sag.

3.6.2 Effect of the span of the Continuity Cable :

Fig.3.11 illustrates the effect of the span of the continuity cable on the unbalance bending moment. Value of ω in the range of 0.2 to 0.25 seem to give small unbalance bending moments. As the value of ω increases the maximum peak unbalance moment decreases but not appreciably. The comparison of the unbalance bending moment for different values of ω is rather difficult because the anchorage point shifts the curve (Fig.3.11) to some extent. However, a change of ω from 0.2 to 0.25 does not alter the results very much.

The unbalance bending moment vary with the relative magnitude of the bending moments caused due to the live load and self weight. The factor r ($r = M_l/M_g$) may vary depending on the span. The value may be as low as 0.5 for long spans and as high as 4 for short spans. Therefore, a set of behaviour and design curves are shown in Figs.3.12 and 3.13. The maximum unbalance bending moment decreases with increase in r . This type of behaviour is advantageous for long span. The effective design moment coefficient for long spans will be of smaller magnitude.

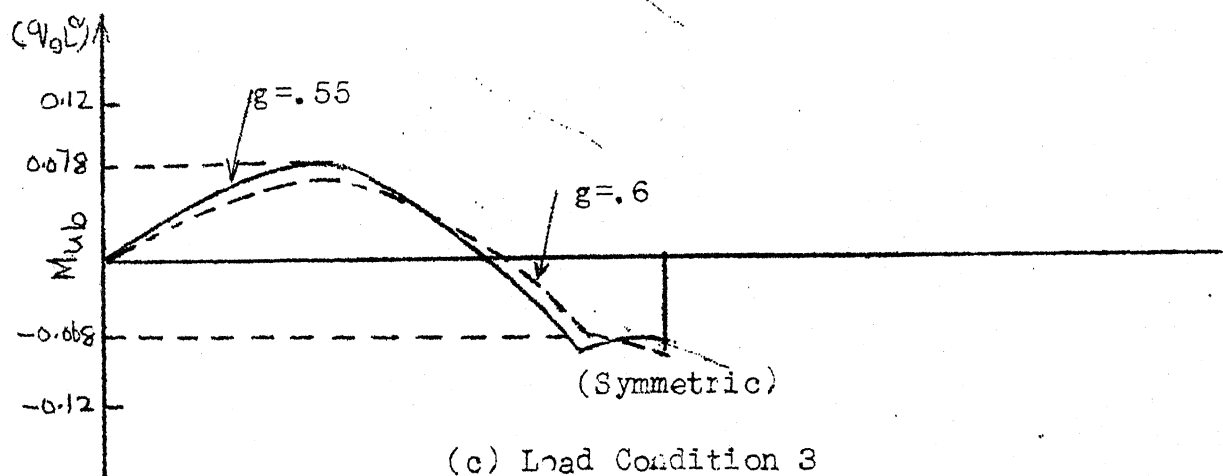
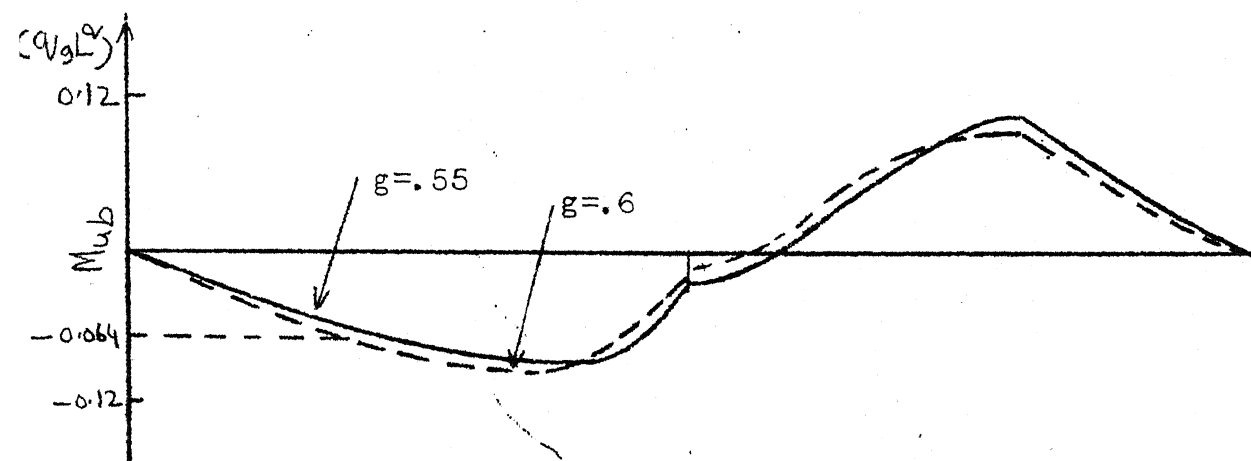
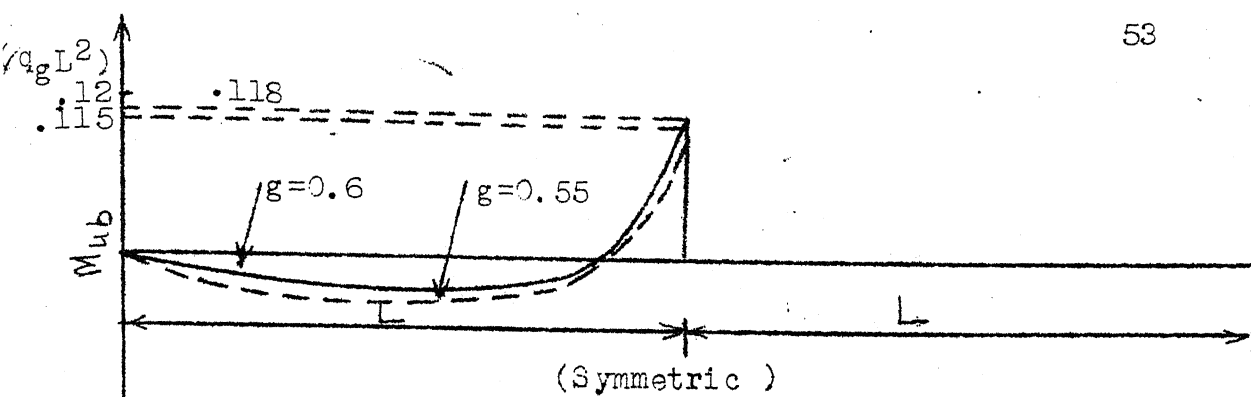


Fig.3.9 Effect of g - Constant ω and r
 $(\omega = 0.15 \text{ and } r = 1.5)$

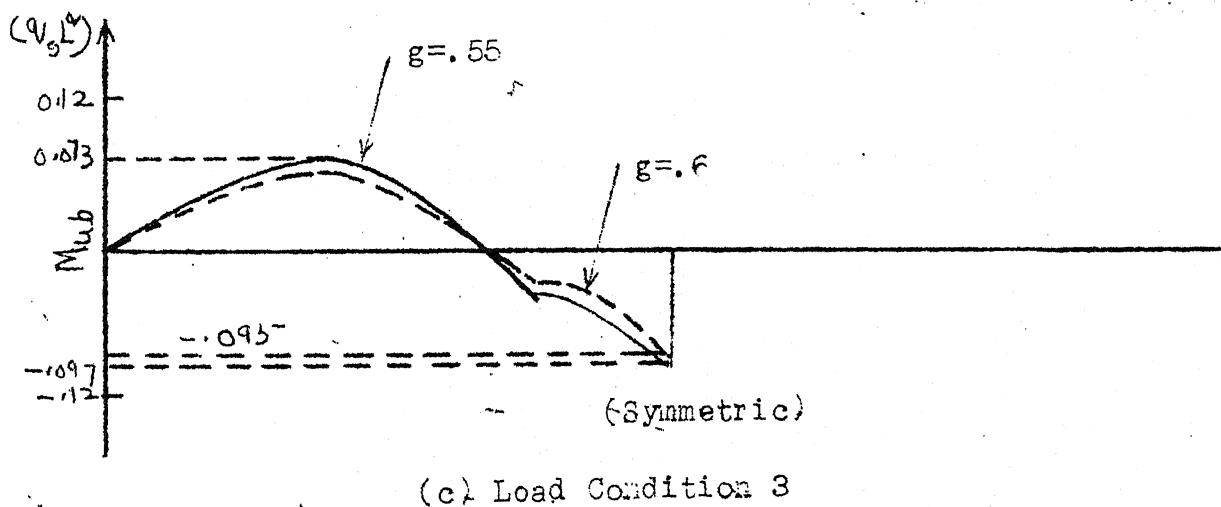
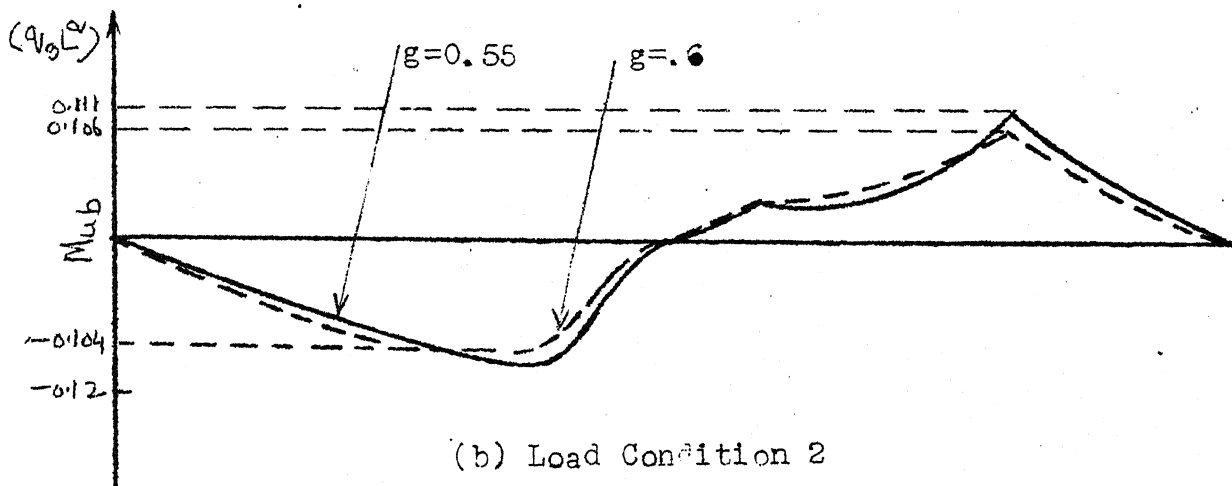
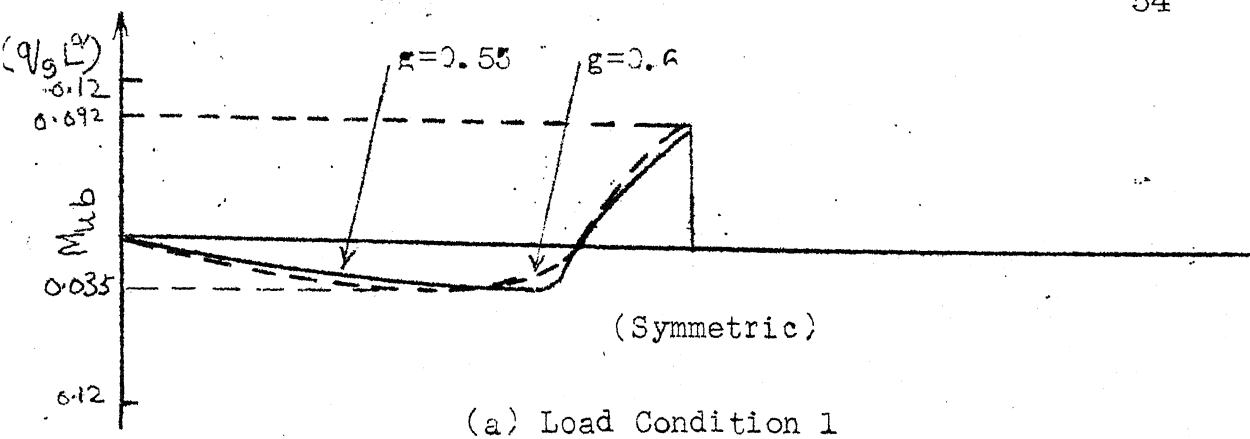


Fig.3.10 Effect of g - Constant ω and r
 $(\omega = .25 \text{ \& } \eta = 1.5)$

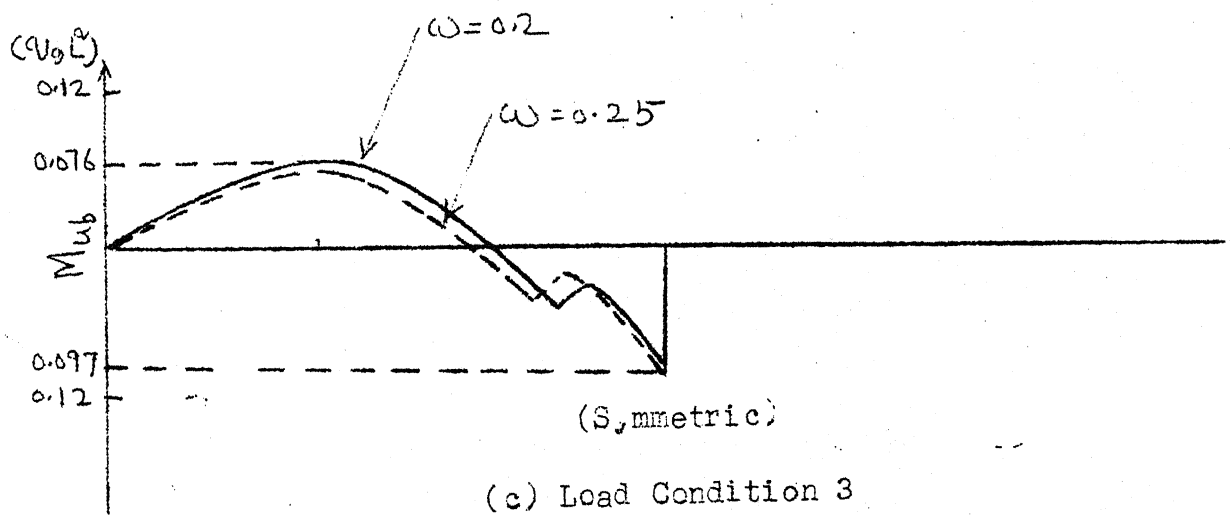
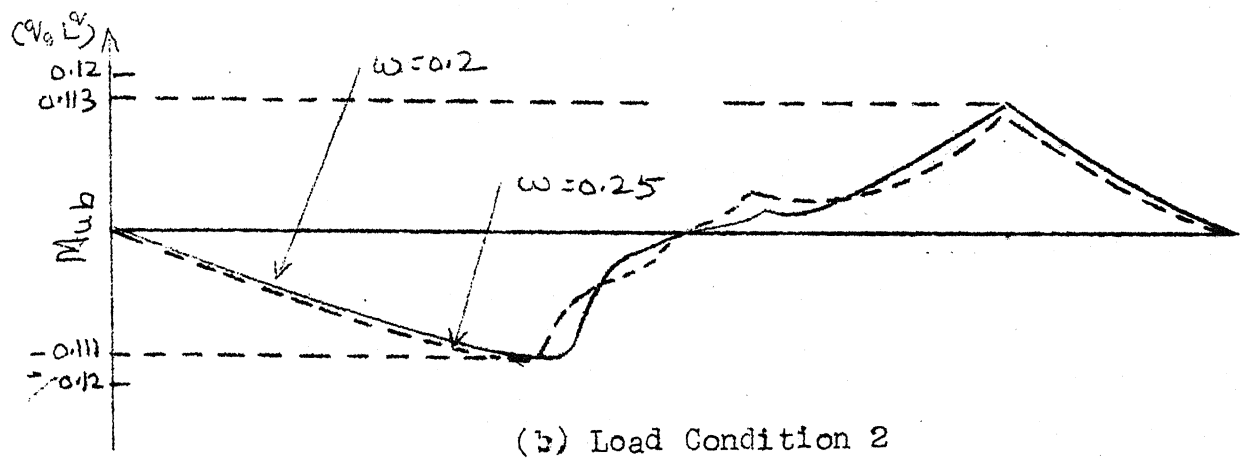
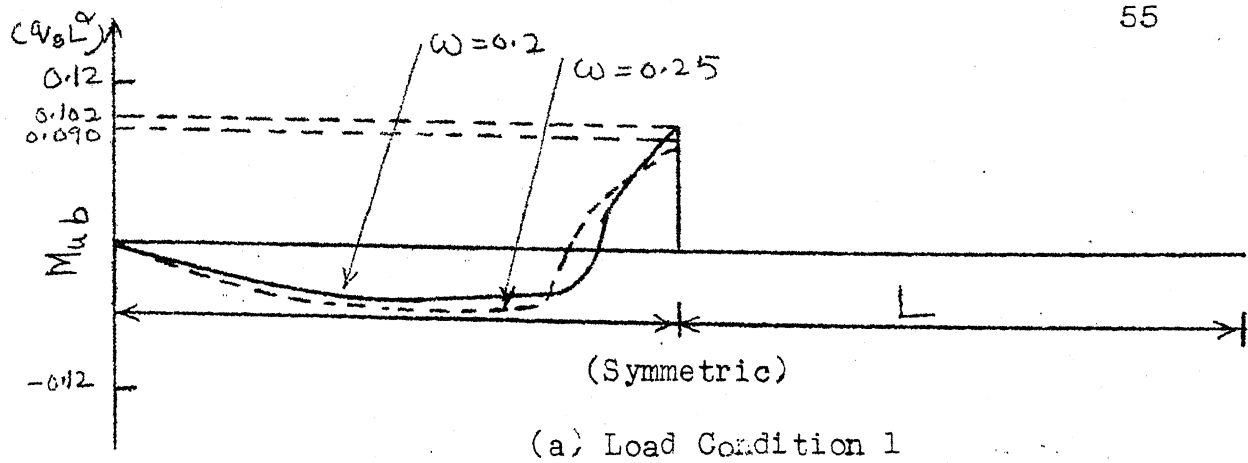
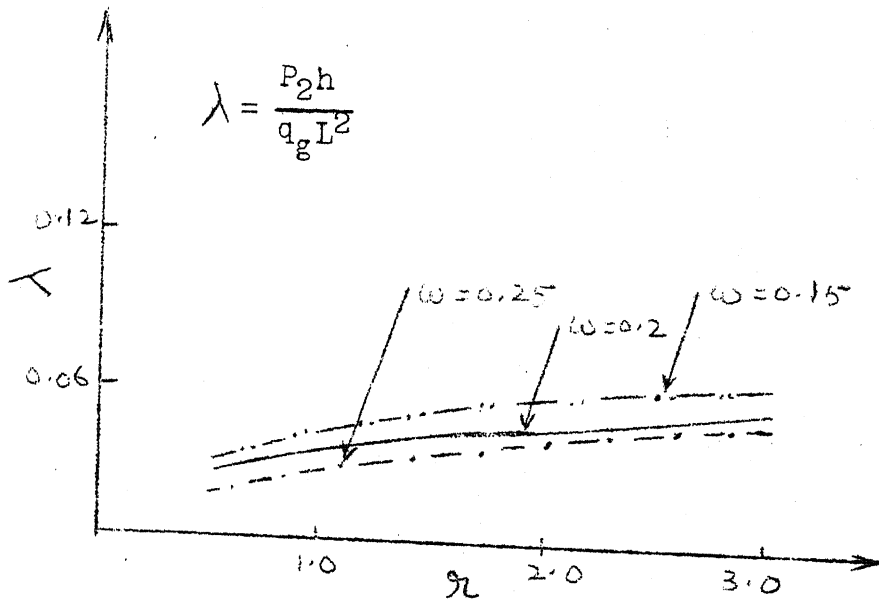
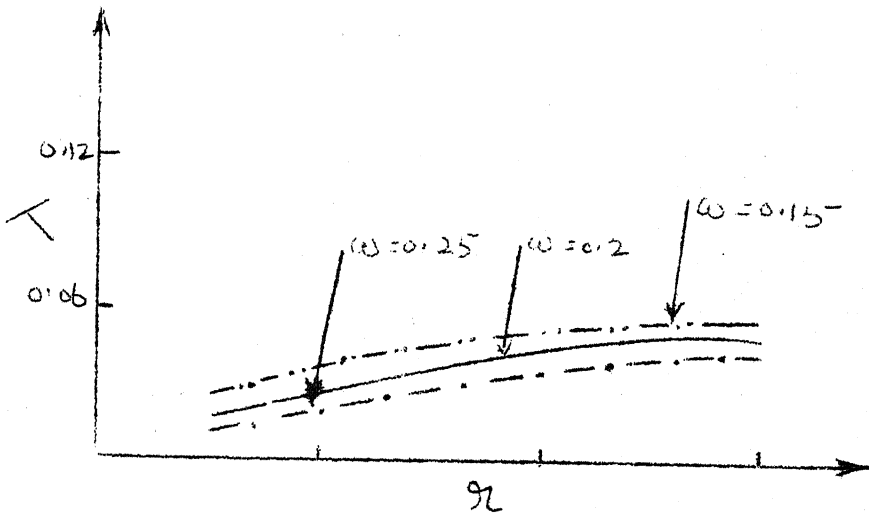


Fig.3.11 Effect of ω - Constant g and r
 $(g = 0.552, r = 1.5)$

$$\lambda = \frac{P_2 h}{q_g L^2}$$



(a) $g = 0.55$



(b) $g = 0.6$

Fig.3.12 Selection of Prestress
(for fixed η and λ of g)

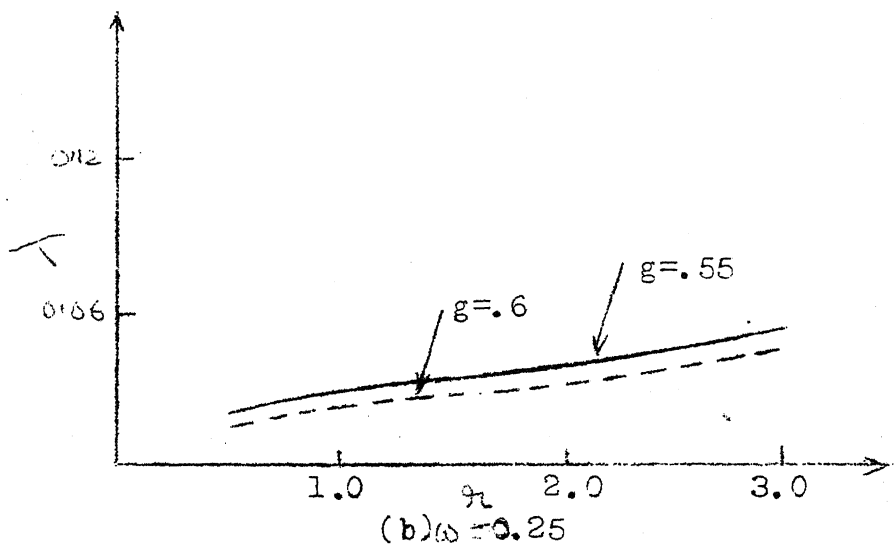
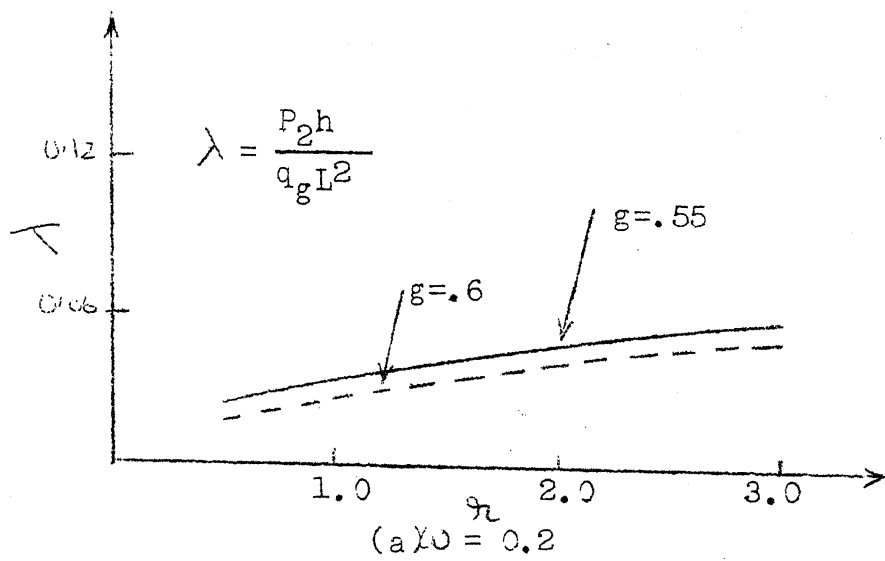


Fig.3.13 Selection of Prestress
(for fixed values of ω)

3.7 Conclusions :

On the basis of investigation carried out the following conclusions can be drawn :

1) The sag (g_h) should be chosen as maximum. This suggests that a simply supported cable profile with maximum deviation at the middle and maximum eccentricity at the support should be provided.

2) The eccentricity of the continuity cable (βh) should be chosen as maximum because maximum eccentricity minimizes the area of steel and maximizes the ultimate moment capacity of the section.

3) There is a definite relation between the two prestressing forces P_1 and P_2 which matches the slopes at the middle support thus making the beam continuous.

4) The effect of the span of the continuity cable (ωL) on the unbalance bending moment is to decrease its magnitude to a certain extent (reduction by a very small percentage).

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